Available online at www.sciencedirect.com
ScienceDirect
Cognitive Systems Research 11 (2010) 243–259
Cognitive Systems RESEARCH
www.elsevier.com/locate/cogsys

Two qubits for C.G. Jung's theory of personality

Action editor: Mark Bickhard

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Received 24 November 2008; accepted 18 November 2009

Abstract

We propose a formalization of C.G. Jung's theory of personality using a four-dimensional Hilbert-space for the representation of two qubits. The first qubit relates to Jung's four psychological functions: **T**hinking, **F**eeling, **S**ensing and i**N**tuition, which are represented by two groups of projection operators, {**T**, **F**} and {**S**, **N**}. The operators in each group are commuting but operators of different groups are not. The second qubit represents Jung's two perspectives of extraversion and introversion. It is shown that this system gives a natural explanation of the 16 psychological types that are defined in the Jungian tradition. Further, the system accounts for the restriction posed by Jung concerning the possible combination of psychological functions and perspectives. The empirical consequences of the present theory are discussed, and the results of a pilot study are reported with the aim to check some basic predictions of the theory. In addition, it is shown why the present praxis of personality diagnostics based on classical statistics is insufficient.

Keywords: Personality theory; Quantum cognition; Qubit; Entanglement; Type indicator; Big five

1. Introduction

Modern personality psychology recognizes persons as complex, multifaceted entities whose understanding requires a whole collection of methods. The field today possesses rich theories and an impressive collection of research methods. Besides the psychodynamic tradition starting with Freud (2000) and continuing with Jung (1921), Adler (1927), Sullivan (1953), and many others, there are influential developments whose inspirations came largely from the general tenets of behaviorism (Cattel, 1943; Eysenck, 1947, 1967; Goldberg, 1993; McCrae & Costa, 1997). We further find socioanalytical theories (e.g. Hogan, 1982), various theories of self-regulation (e.g. Block, 1981; Carver & Scheier, 1981), Tomkins' (1978) script theory, and the life story model of identity (McAdams, 1985; McAdams, 2001).

In this paper, the psychodynamic tradition founded by C.G. Jung is followed. We restrict ourselves to this tradition for two reasons. First, this tradition is still the predominant one in many domains of application, including individual and couples counseling, human resource development, conflict management, interpersonal relationships, negotiating organizational development and team building, and coaching and career planning. The second reason is a methodological one. It concerns the difference between the Jungian tradition and the behaviorist tradition. We have the structural substance of Jungian depth psychology on the one hand, which contrasts with the assumption of a general empiric procedure for detecting the crucial dimensions of human personalities on the other hand. Besides this difference, there is a crucial contrast between the technical prerequisites of the two conceptions. The behaviorist tradition generally assumes Bayesian probabilistics, which is required in order to justify the application of standard statistical methods such as factor analysis and component analysis. In this Bayesian framework, an underlying Bool-

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1389-0417/$ - see front matter
doi:10.1016/j.cogsys.2009.12.002

ean algebraic structure is assumed for modeling events, propositions and (complex) properties. A careful analysis of the original Jungian ideas shows that this assumption is questionable if it comes to consider the needs of an appropriate formalization. Though it is tempting to press the Jungian framework in the Procrustean bed of Bayesian probability theory (Myers-Briggs & Myers, 1980), this is methodologically unsound, and it is important to understand why this is not an adequate way of formalizing the spirit of Jung' personality theory. The body of this paper is devoted to explaining this issue.

For a preliminary illustration of our methodological point, it is useful to consider the question of concepts. According to the classical, set-theoretic picture of concepts, each concept is defined as a set of its instances (Margolis & Laurence, 1999). The system of concepts then can be seen as forming a Boolean algebra. This arises from the fact that basic set-theoretic operations such as complementation, intersection and union are used for constructing new concepts. Hence, if A denotes a concept, then the complement $\neg A$ constitutes another concept. And if A and B denote concepts, then the union $A \cup B$ and the intersection $A \cap B$ both form new concepts. Further, Bayesian probability theory can be developed founded on a Boolean algebra. It is based on a simple axiomatic fact: the additivity of a measure function (probability function) μ: if A and B do not overlap (i.e. $A \cap B \neq \emptyset$), then $\mu(A \cup B) = \mu(A) + \mu(B)$ (Kolmogorov, 1933).

Unfortunately, most *natural* concepts cannot be adequately represented by Boolean algebras, and the idea of Bayesian probabilities is likewise questionable in the context of natural concepts. The reason has to do with the idea of prototypes, as used in cognitive psychology (Margolis & Laurence, 1999). Concepts are formed by the typical exemplars of a set (prototypes). What exemplar does or does not belong to a certain concept depends on the *similarity* between the exemplar and the prototypes that constitute the concept. Mathematically, this idea is described by a Euclidian vector space where the instances are described as vectors and the similarity relation is expressed in terms of the inner product (=scalar product). As a consequence, the set of instances that constitutes a prototype concept can be seen as a *convex* set.[1] Obviously, the domain of convex sets does not form a Boolean algebra: though the intersection of two convex sets is still convex, neither the union of two convex sets nor the complement is convex. Hence, when we see natural concepts as conforming to convex sets, the idea of representing conceptual systems by Boolean algebras breaks down. Likewise, it has been argued that classical probability theory cannot be used for modeling typicality or probability judgments (Aerts & Gabora, 2005; Blutner, 2009; Khrennikov, 2003).

Is there an algebraic structure that describes prototype concepts? Recently, some authors have suggested that the algebraic structure that best fits the idea of prototype concepts is an ortho-algebra (Widdows, 2004a, 2004b; Widdows and Peters, 2003). Interestingly, this kind of structure is underlying quantum logic – a logical foundation of the structure of propositions as formulated in modern quantum physics (Birkhoff & von Neumann, 1936; Dalla Chiara, Giuntini, & Greechie, 2004; Kalmbach, 1983; Piron, 1976). A measure function can also be formulated on an orthoalgebraic basis, but with properties quite different from those of Bayesian probabilities. Some of these properties are investigated in quantum information science (Vedral, 2006).

Taking this and related motivations into account, it is not surprising that an increasing number of authors argue that the basic framework of quantum theory can find useful applications in the cognitive domain (Aerts, Czachor, & D'Hooghe, 2005; Atmanspacher, Römer, & Walach, 2002; Blutner, 2009; Busemeyer, Wang, & Townsend, 2006; Franco, 2007; Khrennikov, 2003; Pothos & Busemeyer, 2009). Recently, Gabora, Rosch, and Aerts (2008) have demonstrated how this framework can account for the creative, context-sensitive manner in which concepts are used, and they have discussed empirical data supporting their view.

The present application of the mathematical framework of quantum theory to personality diagnostics is new. We propose a simple formalization of the crucial traits of C.G. Jung's theory of personality by using the formulation of quantum theory as currently used in the context of quantum information science (Vedral, 2006). Our claim is not only that the structure underlying the diagnostic questions typically asked in personality diagnostics can be characterized by an ortho-algebra. We also aim to demonstrate that concepts like superposition, entanglement, and quantum probabilities are useful instruments for modeling psychodynamic personality theories.

In Section 2 we will present the basic traits of C.G. Jung's theory in some detail. Further, we will refer to three inventories claiming to assess his typology: (a) the Myers-Briggs type indicator, (b) the framework of socionics, and (c) the Singer–Loomis inventory of personality. The discussion will show why the original Jungian framework cannot be pressed in the Procrustean bed of Bayesian probability theory. Further, it demonstrates the potential of the origin Jungian ideas in the context of modern personality theories.

Section 3 introduces some basic concepts of quantum theory including the notion of a qubit and the Pauli spin operators. Section 4 introduces our formal model that addresses the structural ideas of Jungian depth psychology. We also give detailed argumentation as to why we chose this particular approach. Though we have to admit that our model is presently open to several speculations, we give

[1] In Euclidean space, an object is convex if for every pair of points within the object, every point on the straight line segment that joins them is also within the object. For example, a solid globe is convex, but anything that is hollow or has a dent in it is not convex. For better understanding the importance of the notion of convex sets in cognitive science, the reader is referred to Gärdenfors (2000).

some preliminary results of a pilot study empirically testing the model. Section 5 concludes the paper with a general discussion.

Though there are some polemics between and within the different personality schools that follow C.G. Jung, we feel free to ignore these aspects, and to concentrate on approaching a formal model of personality that uses C.G. Jung's ideas in its originally fresh, independent and anti-dogmatic way. Hence, our focus will be on the fruitfulness of certain formal ideas, not on hair-splitting and pedantic justification of various aspects of the offspring of C.G. Jung's theory of personality.

2. C.G. Jung's theory of personality in a nutshell

After almost 20 years of practical experience and work as a specialist in psychiatric medicine, Jung published a remarkable book about psychological types (Jung, 1921). In this book, Jung gave a careful analysis of the universals and differences of human personalities. Jung thought that people were born with an inborn predisposition to type, and that the positive combination of both nature and nurture would see that predisposition expressed healthily. In Jung's theory there are no pure types. There is a set of psychological opposites, equally valuable but realized with different preferences for different personalities. Type preferences themselves are the bridge between the conscious and the unconscious.

In the following, we will demonstrate that Jung's holistic picture of the Self[2] is difficult to reconcile with classical ideas of physical symbol systems (based on Boolean algebras and classical probability theory). Instead, we will argue that a simple quantum theoretical model is sufficient to express the bulk of Jung's theory. Though it is not implausible to assume that Jung felt that the mind and the Self are "resonance phenomena" that are associated with the wave-like aspect of atomic particles, he did not make any attempt to express his theory using the language of quantum mechanics. Developing logically stringent theories was not Jung's strongest talent, and this is perhaps one of the main reasons why Jung was never acknowledged as a forerunner in the unification of psychology, eastern thinking and quantum physics. Regrettably, Jung's cooperation with Nobel Prize Laureate Wolfgang Pauli did not help to lift Jung's informal theory of personality to a more stringent level. Instead, their common reflections were directed far beyond psychology and physics, entering into the realm where the two areas meet, in the philosophy of nature.

Jung basically assumes that all people have roughly the same psychological equipment of apperception and responsiveness. Where people differ is in the way that each of them typically makes use of that equipment. Accordingly, we are confronted with two main questions for the psychologist:

- What are the essential components of the equipment?
- How do people differ in using these components to form their habitual mode of adaptation to reality?

Jung's answer to the first question claims that all people are equipped with four psychological functions, called **T**hinking, **F**eeling, **S**ensing and i**N**tuition, which are realized in one of two different attitudes: **I**ntroversion and **E**xtraversion. Normally, people use all four psychological functions. However, they have different preferences for what functions they use predominantly. Jung claims that it is exactly these differences that constitute the different types of personality.

Jung's typology of personality is pretty popular nowadays, and the introvert-extravert dimension is the most popular part of the theory. We find this dimension in several theories, notably Hans Eysenck's, although it is often hidden under alternative names such as "sociability" and "surgency". Introverts are people who prefer the internal world of their own thoughts, feelings, fantasies and dreams, while extroverts prefer the external world of things, events, people and activities. The words have become confused with ideas like shyness and sociability, partially because introverts tend to be shy and extroverts tend to be sociable. But Jung intended for them to refer more to whether an individual tended to face toward the persona and outer reality, or toward the collective unconscious and its archetypes.

Whether we are introverts or extroverts, we need to deal with the world, inner and outer. And each of us has his preferred ways of dealing with it – ways we are comfortable with and good at. Jung's four basic ways, or functions, are explained now in a bit more detail:

> Ich unterscheide vier Funktionen, nämlich *Empfindung*, *Denken*, *Gefühl* und *Intuition*. Der Empfindungsvorgang stellt im wesentlichen fest, dass etwas ist, das Denken, was es bedeutet, das Gefühl, was es wert ist, und Intuition ist Vermuten und Ahnen über das Woher and das Wohin. (Jung, 1936, p. 270).[3]

Thinking means evaluating information or ideas rationally and logically. Jung called this a *rational* function, meaning that it involves decision making or *judging*, rather than the simple intake of information. **F**eeling, like thinking, is a matter of evaluating information, this time by

[2] Jung's conception of Self should not be confused with Freud's idea of ego (and Superego). Jung sees the Self as the archetype of wholeness and the regulating center of the psyche; a transpersonal power that transcends the ego. As an empirical concept, the Self can be seen as designates the whole range of psychic phenomena in man – including both conscious and unconscious phenomena.

[3] In English translation: "I distinguish four functions, namely *sensation*, *thinking*, *feeling*, and *intuition*. *Sensation* tells us that something exists; *thinking* tells us what it is; *feeling* tells us what its significance is for us; and *intuition* tells us where it comes from and where it is going".

weighing one's overall emotional response. **S**ensing means what it says: getting information by means of the senses. A sensing person is good at looking and listening and generally getting to know the world. Jung called this an *irrational* function, meaning that it involved *perception* rather than judgment of information. **IN**tuiting is a kind of perception that works outside of the usual conscious processes. It is irrational or perceptual, like sensing, but comes from the complex integration of large amounts of information, rather than simple seeing or hearing. Jung said it was like seeing around corners.[4]

Jung says that we all have these psychological functions. We just have them in different proportions. Each of us has a *superior* function, which we prefer and which is best developed in us, a *secondary* function, which we are aware of and use in support of our superior function, a *tertiary* function, which is only slightly less developed but not particularly conscious, and an *inferior* function, which is poorly developed and so unconscious that we might deny its existence in ourselves.

Across the different types of personality, there are several restrictions that determine which functions can be realized under what attitude at what position in the rank ordering of the functions. To understand these restrictions, it is important to see that the functions are organized as equally valuable psychological opposites. **T**hinking and **F**eeling constitute a pair of opposites [rational opposites], as do the pair **S**ensation/i**N**tuition [irrational opposites].

> Der Denktypus zum Beispiel muss notwendigerweise immer das Gefühl möglichst verdrängen und ausschließen, weil nichts so sehr das Denken stört wie das Gefühl, und umgekehrt muss der Fühltyp das Denken tunlichst vermeiden, denn nichts ist dem Gefühl schädlicher als das Denken (Jung, 1923).[5]

Hence, the first restriction is that if the superior function is rational/irrational, then the secondary function must be irrational/rational. Otherwise the secondary function cannot support the superior function. Plausibly, the alternation of rational and irrational functions is continued along the ranking hierarchy.

In order to convey his idea of how the four functions work together, Jung offered the image of a cross. Fig. 1 shows an image which is a slight modification of the original picture that can be found in Jung, von Franz, and Henderson (1968, p. 60). As Fig. 1 makes pretty clear, there are only two options in each case for fixing the secondary function, and this is exactly the content of the first restriction mentioned above.

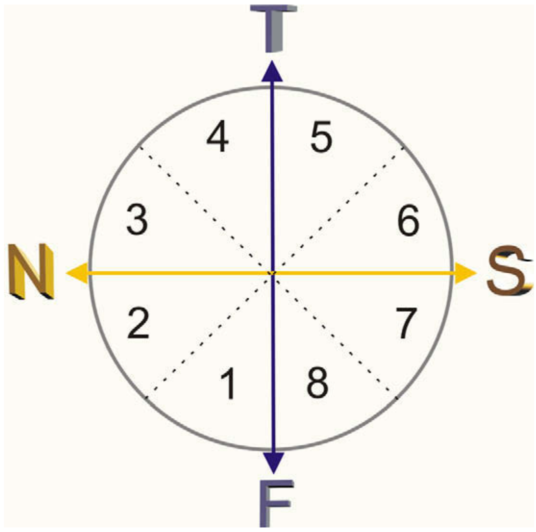

Fig. 1. Two pairs of opposite psychological functions: **T**hinking and **F**eeling [rational opposites], **S**ensation/i**N**tuition [irrational opposites]. Jung takes this two-dimensional representation in order to demonstrate the dependencies between the psychological functions. For example, **T**hinking and **F**eeling are opposites and conflict with each other (assuming one fixed attitude). However, their effect can be modified by using the irrational functions (**S**ensation and i**N**tuition, respectively). The numbers enumerate eight sectors in dependence of what are the two dominant functions (see Table 1 and the associated text for more explanations).

There is a second restriction which constrains the attitudes that can be connected with the different psychological functions:

> Neben diesen eben erwähnten Qualitäten der unterentwickelten Funktionen kommt letzteren auch die Eigentümlichkeit zu, dass sie bei bewusster introvertierter Einstellung extrovertiert sind und umgekehrt, dass sie also zugleich die bewusste Einstellung kompensieren. Man darf daher erwarten, bei einem introvertierten Intellektuellen zum Beispiel extravertierte Gefühle zu entdecken. (Jung, 1923).[6]

Hence, the restriction is that opponent (or *dual*) functions have different attitudes. Possibly the rationale for this restriction is related to the evolution of typologies. As Jung mentions in chapter X of his book "Psychologische Typen" (Jung, 1921), the personality of an individual conflicts if the Self has to realize two opponent functions in the same atti-

[4] The *rational* and *irrational* descriptions that Jung attached to the four functions can lead to misunderstandings, especially given that Jung's use of the words is rather different to the modern meanings. For that reason the characterization of the first group as *judging* dimension and the second group as *perceiving* dimension could be seen as more appropriate. In the following, however, we will further use the traditional terms *rational/irrational*.

[5] In English translation: "The thinking type, for instance, has to necessarily repress and eliminate feeling, because it is feeling that interferes the most with thinking. On the contrary, feeling type has to urgently avoid thinking, because nothing is more harmful to feeling than thinking."

[6] In English translation: "Apart from these aforementioned qualities of the inferior functions, the latter also possess the feature that they are extraverted assuming a conscious introverted attitude (and vice versa), which means that they compensate their conscious attitude at the same time. Consequently, one can expect, for example, to discover extraverted feelings in an introverted intellectual."

tude. There is an evolutionary pressure to avoid such constellations. This pressure results in a kind of dynamics that is called type dynamics.[7]

Obviously, the two restrictions do not constrain the free choice of the superior function, and, consequently, Jung proposed eight basic psychological types, four with the extraverted attitude and four with the introverted attitude: E/I **F**eeling type (corresponding to the regions 1 and 8 in Fig. 1), E/I i**N**tuition type (regions 2 and 3), E/I **T**hinking type (regions 4 and 5), and E/I **S**ensing type (regions 6 and 7). These eight basic types discussed by Jung can be further refined into 16 psychological types depending on what is considered the secondary function.

Jung further introduces the concept of "shadow" in explaining the individuation process. By shadow he means the "negative" side of the personality, the sum of all those unpleasant qualities we like to hide. Integrating the shadow and the inferior cognitive modes that reside there into the overall cognitive functioning is an important step of the individuation process.

Notably, there are three different systems that make use of the 16 types. The first system is the *Myers-Briggs type indicator* (MBTI). The second system is the scheme of *socionics*. The third and possibly the most developed system is the *Singer–Loomis inventory of personality* (SLIP).

MBTI was developed by Katharine Cook Briggs and her daughter, Isabel Briggs Myers, based on a roughly simplified picture of C.G. Jung's ideas (Myers, 1962; Myers & McCaulley, 1985; Myers-Briggs & Myers, 1980). The MBTI classifies a person's personality along four dichotomous categories. In each case, the emphasis is on an either-or preference (somewhat akin to your preference for being either right or left handed). In the MBTI, the first element indicates the preferred attitude (**E**xtraverted/**I**ntroverted), the second element indicates the preferred irrational function – whether you tend to take in new information as it is (**S**ensing) or connect it with ideas of what could be (i**N**tuition), the third indicates the preferred rational function – whether you value emotions and values over logic and reason (**F**eeling) or the other way around (**T**hinking), and the fourth element indicates whether the rational function is more important than the irrational one, i.e. whether you prefer planned order and quick decisions (**J**udging) or spontaneity and contemplation (**P**erceiving).[8]

Socionics was developed in the 1970s and 1980s mainly by the Lithuanian researcher Aušra Augustinavičiūtė. The name 'socionics' is derived from the word 'society', since Augustinavičiūtė believed that each personality type has a distinct purpose in society, which can be described and explained by socionics. The system of socionics is in several respects similar to the MBTI; however, whereas the latter is dominantly used in the USA and Western Europe, the former is mainly used in Russia and Eastern Europe. For more information, the reader is referred to the website of the International Institute of Socionics and to several scientific journals edited by this institution (see http://www.socionics.ibc.com.ua/).

Despite of several similarities there are also important differences. For instance, the MBTI is based on questionnaires with so-called forced-choice questions. Forced-choice means that the individual has to choose only one of two possible answers to each question. Obviously, such tests are self-referential. That means they are based on judgments of persons about themselves. Socionics rejects the use of such questionnaires and is based on interviews and direct observation of certain aspects of human behavior instead. However, if personality tests are well constructed and their questions are answered properly, we expect results that often make sense. For that reason, we do not reject test questions principally, but we have to take into account their self-referential character. Another difference relates to the fact that socionics tries to understand Jung's intuitive system and to provide a deeper explanation for it, mainly in terms of informational metabolism (Kepinski & PZWL, 1972). Further, socionics is not so much a theory of personalities *per se*, but much more a theory of type relations providing an analysis of the relationships that arise as a consequence of the interaction of people with different personalities.

The 16 psychological types correspond to the 8 sectors in Fig. 1 if we take into account that the two dominant (conscious) psychological functions can be either in the extraverted attitude or in the introverted attitude. Table 1 gives the complete classification in a system based on the first two dominant functions and in the closely related type indicator developed by Myers-Briggs (MBTI).

Here are some examples illustrating the types of forced-choice questions used in the empirical type test à la Myers-Briggs:

Table 1
16 Psychological types. Following the Jungian tradition, the first two psychological functions are given with the corresponding attitude (extraverted/ introverted). Further, the closest pendant in the MBTI is specified.

Extravert			Introvert		
1	1EF 2EN	ENFJ	1IF 2IN	INFP	1
2	1EN 2EF	ENFP	1IN 2IF	INFJ	2
3	1EN 2ET	ENTP	1IN 2IT	INTJ	3
4	1ET 2EN	ENTJ	1IT 2IN	INTP	4
5	1ET 2ES	ESTJ	1IT 2IS	ISTP	5
6	1ES 2ET	ESTP	1IS 2IT	ISTJ	6
7	1ES 2EF	ESFP	1IS 2IF	ISFJ	7
8	1EF 2ES	ESFJ	1IF 2IS	ISFP	8

[7] See, for instance, Loomis (1991); see also the following website: http://en.wikipedia.org/wiki/Myers-Briggs_Type_Indicator#cite_ref-Myers_1-4/index.html.

[8] Myers believed that the first extraverted function governs the choice between **P**erceiving and **J**udging, in contrast to C.G. Jung, who related this to the first function (without considering the qualification as extraverted or introverted). Consequently, the MBTI is using the reverse order of the first two functions in cases of introverted people. Socionics criticises this point.

(1) **E**xtraverted/**I**ntroverted opposition
(a) At parties, do you stay late with increasing energy or leave early with decreased energy? (E/I)
(b) In doing ordinary things are you are more likely to do it the usual way, or do it your own way? (E/I)
(c) When the phone rings, do you hasten to get to it first, or do you hope someone else will answer? (E/I)
(2) **F**eeling/**T**hinking opposition
(a) In making decisions do you feel more comfortable with feelings or standards? (F/T)
(b) In approaching others is your inclination to be personal or objective? (F/T)
(c) In order to follow other people do you need trust, or do you need reason? (F/T)
(3) **S**ensing/i**N**tuition opposition
(a) Which seems the greater error: to be too passionate or to be too objective? (S/N)
(b) Are you more attracted to sensible people or imaginative people? (S/N)
(c) Facts speak for themselves or illustrate principles? (S/N)

Taking C.G. Jung's theory seriously, the expression of a person's psychological type is more than the sum of the four individual preferences expressed by the MBTI. This is because of the way in which the preferences interact through type dynamics and type development. Although the interpretation of the MBTI acknowledges the role of type dynamics and type development, these concepts do not enter the test procedure. As an example, assume that for a person X, the test results indicate a perfect balance between **E**xtraversion and **I**ntroversion (i.e. 50% E, 50% I). Assume further that we also find a perfect balance between **T**hinking and **F**eeling and a low percentage of the irrational functions. Obviously, the type of *extraverted thinkers* and the type of *introverted thinkers* are both in agreement with this test results. Unfortunately, there is no way to discriminate the two types by simply testing the percentage of E, I, T and F. The reason is that, due to type dynamic in the case of extraverted thinkers, the ET function is superposed with the IF function. And in the case of introverted thinkers the IT function is superposed with the EF function. In both cases we get 50% E, 50% I, 50% F and 50% T. Hence, though there is a big difference between the personality types that agree with the test results, there is no way to determine the correct Jungian type by using the MBTI. What we need are particular test questions that directly test for the functions in a certain attitude.

More recently, Singer, Loomis, and their followers (Loomis, 1991; Loomis & Singer, 1980; Loomis, 1987; Myers, 1962; Myers & McCaulley, 1985; Singer & Loomis, 1984) have criticized several aspects of the MBTI, including the fact that this test: (a) does not really include Jung's claim that the psychological functions cannot be considered in isolation, but always with respect to a definite attitude, (b) does not regard quantitative interpretation of the MBTI scores,[9] and (c) assumes bipolar opposites of psychological functions rather than logically independent response items corresponding to the two poles of the "opposites".

With regard to the latter point, Loomis and Singer argued that

> If Jung's bipolarity assumption was correct, that is, if the oppositional arrangement of the functions was inherently within the individual psyche, then it was not necessary to use forced-choice questions to assess an individual's typology. On the other hand, if Jung's bipolar assumption was not valid, changing the forced-choice items in the GW and the MBTI to another format would have an effect on the resulting profiles." (Loomis, 1991, p. 45)

Hence, instead of one forced choice question such as (2b), repeated here for convenience, two independent response items such as (2b′) and (2b″) were presented:

(2)(b) In approaching others is your inclination to be personal or objective?
(b′) In approaching others is your inclination to be personal?
(b″) In approaching others is your inclination to be objective?

(2)

This change of the procedure changed the outcomes drastically. For instance, from 79 subjects of the MBTI study (modifying the original MBTI material) 36 (46%) were found who did not have the same superior function on the revised version as they had on the original version and 29 (36%) were found who did not have the superior function opposed to the inferior function (Loomis & Singer, 1980).

Though Loomis and Singer, as well as the school of Socionics, generally stress the consideration of psychological functions as always being relative to a certain attitude, extraverted or introverted, this crucial point was disregarded in the Loomis & Singer (Loomis & Singer, 1980) experimental study. Possibly, this could explain the surprising result. In forced-choice questions for both parts of the question – "be personal" or "be objective" in example 2 – the same attitude is taken. However, when two isolated questions are constructed, then it is possible that different attitudes are taken for the two questions: For example, (2b′) could be interpreted as highlighting introverted feeling, connecting us to our inner values. And (2b″) could be interpreted as highlighting extraverted thinking, connecting us to the outer, physical world. In contrast, when asking (2b), one and the same attitude is taken, though it is not always clear which attitude. Assuming that the extraverted attitude is more probable,

[9] "Scores were designed to show the direction of a preference, not its intensity." (Myers & McCaulley, 1985, p. 58).

we get a preference for the answer "objective". Hence, it could be that our superior function is **T**hinking, and **F**eeling is the inferior one. This contrasts with the revised case of two isolated questions where both **T**hinking and **F**eeling could be the two highest ranked psychological functions. In this way, the results of the Loomis and Singer (1980) study become understandable.

For good reasons C.G. Jung was relatively vague concerning the details of type ranking and type dynamics. However, he seems to claim that the auxiliary function has the same attitude as the first function (otherwise the auxiliary function could not support the first function),[10] and he insists on claiming that the unconscious inferior function always has the opposite attitude of the superior function.[11] Recent experimental studies made use of the full inventory of the Singer–Loomis type development inventory and clearly demonstrated the shortcomings of the MBTI. Unfortunately, these studies were not really conclusive about empirical constraints concerning possible sequences of psychological functions in a certain attitude and other aspects of type dynamics (Dugan & Wilson, 2002; Vacha-Haase & Thompson, 2002; Wilson, Dugan, & Buckle, 2002).

3. Basic concepts of quantum theory

Modern quantum theory is a conceptual framework relating *states* and *observables* in a dynamic way. States describe aspects of the physical world, and observables relate to meaningful questions we can ask about the world. States are modeled within a vector space. This idea tells us that states can enter into certain systematic operations. For instance, we can add two existing states resulting in a sum vector. The addition of states relates to the phenomenon of superposition. Superposition is mainly known from the physics of waves. However, it also applies in the cognitive domain where we find superpositions of percepts, such as in the domain of colors, faces, and music see, for example (Gärdenfors, 2000). Further, we can multiply a vector with a scalar. That means that we lengthen or shorten the relevant vector. However, things are more complicated if we take the wave-like character of physical objects into account. Then we also need an operation to change the phase of the corresponding wave. This is described mathematically by a scalar multiplication with a complex number of unit length. Such complex scalars are usually written as $e^{i\Delta}$ where angle Δ describes the corresponding phase shift.

Another aspect of the idea of modeling states by vectors is the desire to determine the similarity of two states. The relevant operation is the so-called scalar product. The scalar product of two vectors u and v relates to the product of the lengths of the two vectors multiplied with the *cosine* of the angle between the two vectors. If this angle is $\pi/2$ we will say the two vectors are orthogonal to each other – meaning that they are maximally dissimilar. Formally, states are described as elements of a vector space with a defined metrics. In the general case such a structure is called a Hilbert space. A Hilbert space $\mathscr{H}$ is a (complete) complex vector space upon which an inner product (=scalar product) is defined. The scalar product of two vectors u, v in $\mathscr{H}$ is written in the form $\langle u|v\rangle$. We assume some familiarity with the notion of a vector space and an inner product. For details, the reader is referred to introductory textbooks in quantum information science, e.g. Vedral (2006).

In the following, we will make use of finite Hilbert spaces only, i.e. Hilbert spaces which are spanned by a finite system **S** of linearly independent vectors, which can be assumed to be pairwise orthogonal, i.e. the scalar product of two different vectors in **S** is zero.

In quantum theory, *observables* are modeled by "normal" linear operators of the Hilbert space. Intuitively, such observables ask about the value of a certain real-valued variable, e.g. what is the momentum/energy/place/... of a particle? The value also can be discrete, e.g. 1, standing for *yes,* −1, standing for no, and 0, standing for *do not know*. The expected, averaged answer to the question asked by an observable **a** in a certain state u is formally expressed by the scalar product between u and the state $a|u\rangle$ that results by applying the operator a to the state $|u\rangle$. Hence, for the expected answer we can write

$$\langle \mathbf{a}\rangle_u = \langle u|\mathbf{a}|u\rangle \tag{4}$$

The expected answer may differ from the real answer given after performing the relevant experiment. However, if the experiment is repeated a sufficient number of times, the expected answer reflects the average of the real answers given in exact replications of the experiment.

Observables are often uncertain when measured in a certain state. The root mean square deviation (=standard deviation) is the standard mathematical measure for calculating the uncertainty of an observable in a given state. It is defined as follows:

$$\Delta_u(\mathbf{a}) = \sqrt{(\langle \mathbf{a}^2\rangle_u - \langle \mathbf{a}\rangle_u^2)} \tag{5}$$

For each physical observable there are some states where the answer is absolutely certain, i.e. where the standard deviation is zero. These states are called 'eigenstates' of the operator. In such states, the application of the operator to the states results in a scalar multiplication of the state. In can be shown that under very general conditions each operator has exactly n orthogonal eigenstates where n is the dimension of the underlying Hilbert space (spectrum theorem).

[10] In his concise description of C.G. Jung's theory of personality Stevens (1994) presents a picture (Fig. 2) that exactly makes this point and demonstrates that the first two psychological functions have the opposite attitude as the last two functions.

[11] The MBTI seems to deviate from this position in assuming that the auxiliary function has the opposite attitude as the superior function, hence ESTJ would conform to the following four functions: 1 ET 2IS 3N 4IF and INFP, to take the shadow example, is 1IF 2EN 3S 4ET. In our opinion, it is extremely difficult to empirically justify these details. That does not exclude the possibility that a good theory leads to a decision at this point making speculations superfluous.

If you have more than one observable, then an important question is whether there are states where a simultaneous measurement of these observables can lead to a definite, absolutely certain result. The famous answer given by Heisenberg is that states with definite values for both observables, say **a** and **b**, exist if and only if the ordering of the two observables does not matter, i.e. **ab**–**ba** = 0. For example, states with definite position **X** and simultaneously definite momentum **P** do not exist in quantum mechanics. In this case, a non-zero canonical commutation relation applies: **XP**–**PX** = $ih/2\pi$, where h is the Planck constant.

The size of this ordering effect determines a lower bound for the product of the uncertainties of **a** and **b**. This is the content of the famous Heisenberg uncertainty principle[12]:

$$\Delta_u(\mathbf{a})\Delta_u(\mathbf{b}) \geq 1/2\langle \mathbf{ab} - \mathbf{ba}\rangle_u \tag{6}$$

In the case of position and momentum, the predicted lower boundary is $h/4\pi$.

The simplest non-trivial physical system is a two-state system, also called a *qubit*. In such a system each proper observable has exactly two (orthogonal) eigenvectors, i.e. two states where the question asked by the corresponding observable has a certain outcome. Of course, a qubit contains an infinite set of states, but only two states relate to eigenstates of an observable.

Formally, an arbitrary state of a qubit can be written as

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad \text{with} \quad \alpha^2 + \beta^2 = 1 \tag{7}$$

Here, the two states $|0\rangle$ and $|1\rangle$ are two orthogonal unit vectors of our two-dimensional Hilbert space. In such a physical system, exactly three independent observables are possible. A common choice for these operators is the Pauli operators σ_x, σ_y, and σ_z, which are defined in terms of the operators $|u\rangle\langle v|$ with unit vectors u and v[13]:

$$\begin{aligned} &\text{a.} \quad \sigma_x = |1\rangle\langle 0| + |0\rangle\langle 1| \\ &\text{b.} \quad \sigma_y = i|1\rangle\langle 0| - i|0\rangle\langle 1| \\ &c. \quad \sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1| \end{aligned} \tag{8}$$

Examples for realizing qubits are the spin of electrons (here the three operators give the spin in directions x, y and z), or the polarization of photons (here σ_z is measuring the polarization in ↑-direction, σ_x the polarization in ↗-direction, and σ_y is measuring left and right circularly polarized light). The eigenvectors of σ_z are $|0\rangle$ and $|1\rangle$, with eigenvalues 1 (*yes*) and –1 (*no*). These contrast with the eigenvectors of σ_x, namely $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, which are simple superpositions of the base states. And the eigenvectors of σ_y are superpositions of the base states including a $\pi/2$ phase shift: $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$ and $\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$.

[12] See Appendix A for a simple proof.
[13] The definition of this elementary operator is $|u\rangle\langle v|\,(|w\rangle) =_{\text{def}} |u\rangle \cdot \langle v|w\rangle$ for each state $|w\rangle$ of the Hilbert space.

Making use of a particular parameterization of the states $|\psi\rangle$, every state of a qubit can be realized as the point on a three-dimensional sphere, the so-called Bloch sphere.

$$|\psi\rangle = \cos(\theta/2)e^{-i\Delta/2}|0\rangle + \sin(\theta/2)e^{+i\Delta/2}|1\rangle \tag{9}$$

The parameters θ and Δ are nothing other than spherical polar coordinates, $0 \leqslant \Delta < \pi$ and $-\pi \leqslant \theta < \pi$.

For a simple illustration, consider a photon in a qubit state $|\psi\rangle$, and take $|0\rangle$ as indicating horizontal polarization, and $|1\rangle$ as indicating vertical polarization. Then the probability that the object is horizontally polarized (i.e. it collapses into the state $|0\rangle$) is

$$\langle \mathbf{0}\rangle_\psi = |\langle 0|\psi\rangle|^2 = \cos^2(\theta/2) = 1/2(1 + \cos(\theta)) \tag{10}$$

And the probability that it is vertically polarized (i.e. it collapses into the state $|1\rangle$) is

$$\langle \mathbf{1}\rangle_\psi = |\langle 1|\psi\rangle|^2 = \sin^2(\theta/2) = 1/2(1 - \cos(\theta)) \tag{11}$$

Further, we also can calculate the probability that the object is polarized into a direction given by the eigenvectors of σ_x, namely $|\nearrow\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ or $|\nwarrow\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$$\begin{aligned} &\text{a.} \quad \langle \nearrow\rangle_\psi = 1/2|\langle 0 + 1|\psi\rangle|^2 = 1/2(1 + \sin(\theta)\cdot\sin(\Delta/2)) \\ &\text{b.} \quad \langle \nwarrow\rangle_\psi = 1/2|\langle 0 - 1|\psi\rangle|^2 = 1/2(1 + \cos(\theta)\cdot\sin(\Delta/2)) \end{aligned} \tag{12}$$

In this case the calculated probability also depends on the phase shift Δ. Finally, it is straightforward to calculate the corresponding expectations for the Pauli operators in state $|\psi\rangle$:

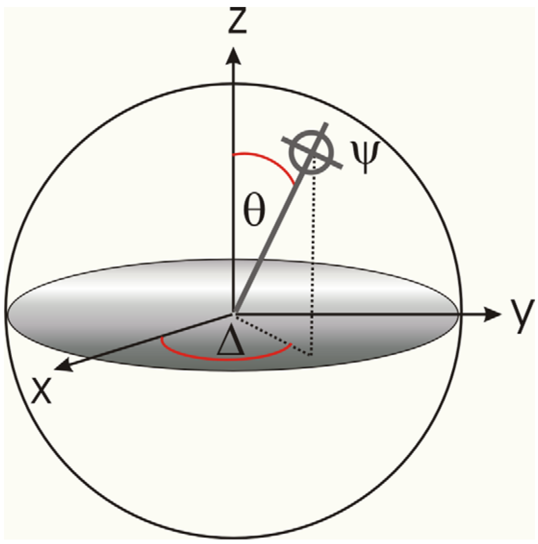

Fig. 2. Bloch sphere. Using Eq. (10) an arbitrary (normalized) state of the two-dimensional Hilbert space can be parameterized by the two spherical polar coordinates θ and Δ. Here, Δ corresponds to a phase shift of the two superposing states $|0\rangle$ and $|1\rangle$).

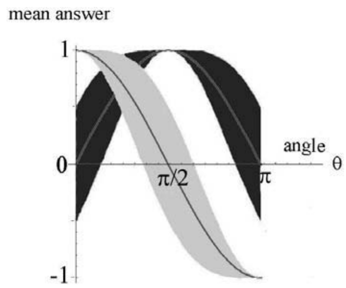

Fig. 3. Expectation values for the complementary observables σ_x (dark) and σ_z (bright) in case of a qubit states with zero phase shift Δ. The graph also indicates the corresponding standard deviations.

$$\begin{aligned} &\text{a.}\quad \langle\sigma_x\rangle_\psi = \sin(\theta)\cdot\cos(\Delta) \\ &\text{b.}\quad \langle\sigma_y\rangle_\psi = \sin(\theta)\cdot\cos(\Delta) \\ &\text{c.}\quad \langle\sigma_z\rangle_\psi = \cos(\theta) \end{aligned} \tag{13}$$

In cases of states with no phase shift between the two components $|0\rangle$ and $|1\rangle$, i.e. where $\Delta = 0$, we see that the operator's σ_y expectations are always zero and the expectations for σ_x and σ_z are $\sin(\theta)$ and $\cos(\theta)$, respectively. It is interesting to calculate the standard deviation $\Delta_\psi(\sigma_x)$ and $\Delta_\Psi(\sigma_z)$ in this case:

$$\begin{aligned} &\text{a.}\quad \Delta_\psi(\sigma_x) = |\cos(\theta)| \\ &\text{b.}\quad \Delta_\psi(\sigma_z) = |\sin(\theta)| \end{aligned} \tag{14}$$

Fig. 3 shows the expectation values for the complementary observables σ_x and σ_z, including an indication of the corresponding standard derivations.

The picture clearly demonstrates the complementary character of the observables σ_x and σ_z: there is no state where a simultaneous measurement of both observables is possible without any uncertainty. Instead, if one observable can be measured without deviation, the other one can be measured with maximal uncertainty only.[14]

In quantum theory, complementary observables are in a sharp contrast to opponent observables. A pair of opponent observables makes a precise simultaneous measurement possible, but the values of the two observables are opponent in these cases (e.g. +1 for one observable and −1 for the other, and vice versa). In the present case of Pauli operators we can simply derive the opponent counterparts of the Pauli observables by multiplying them with −1 (corresponding to a phase shift of $\Delta = \pi$). Fig. 4 illustrates the opponent observables σ_z and $-\sigma_z$. Evidently, at $\theta = 0$ and $\theta = \pi$ the measurement is sharp and the results are +1, −1 and −1, +1, respectively.

As with classical bits it is possible to put qubits together in order to build and store more information. In quantum theory complex systems are constructed by using tensor products ⊗. This operation applies both to vectors of the Hilbert space $|u\rangle \otimes |v\rangle$ and to linear operators $\mathbf{a} \otimes \mathbf{b}$. If the context precludes misunderstandings, it is convenient to leave out the ⊗. Hence we will write $|0\ 1\ 1\rangle$ instead of $|0\rangle\otimes|1\rangle\otimes|1\rangle$ and **0 1 1** instead of $\mathbf{0}\otimes\mathbf{1}\otimes\mathbf{1}$.

In quantum theory, the existence of entangled states of several qubits is of the greatest importance. In such entangled states, a single qubit does not have a definite state. However, the system of the qubits (as a whole) is in a definite state. This can be tested by fixing the first qubit (by a local measurement). Then the result of measuring the second qubit is always definite, i.e. 100% predictable. This leads to so-called nonlocal effects as described in the EPR experiment (Einstein, Podolsky, & Rosen, 1935), at least in the standard physical case where the underlying elementary objects have a spatial distribution such as single electron spin qubits or single photon polarization qubits.

Recently, several researchers have mentioned the cognitive relevance of quantum theory (Aerts et al., 2005; Atmanspacher et al., 2002; Busemeyer et al., 2006; Conte et al., 2008; Franco, 2007; Khrennikov, 2003; Pothos & Busemeyer, 2009). One argument has to do with the nature of concepts as investigated in Cognitive Science. As mentioned in Section 1, *natural* concepts cannot be adequately represented by Boolean algebras. Instead, natural concepts form convex sets (Gärdenfors, 2000). The class of convex sets is preserved by a rich variety of algebraic operations including set intersection, summation, and orthogonal complementation (Rockafellar, 1970). Though we cannot go into any details here, it should be remarked that the algebra of convex sets resembles an ortho-algebra that is used to model propositions and yes/no questions in quantum theory (Hamhalter, Navara, & Pták, 1995). The main arguments, however, that are put forward by the authors mentioned before has to do with the order-dependence of questions and interference effects found in simple decision tasks (Blutner, 2009). The effect of entanglement is much less spectacular in the cognitive domain since the underlying 'objects' do not have any spatial organization. As we

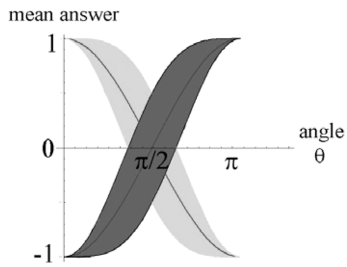

Fig. 4. Expectation values for the opponent observables σ_z (bright) and $-\sigma_z$ (dark) in case of a qubit state with zero phase shift Δ including an indication of the corresponding standard derivations.

[14] The standard uncertainty principle (6) repeated here in the case of the observables σ_x and σ_z.

$$\Delta_\psi(\sigma_x)\Delta_\psi(\sigma_z) \geq 1/2\langle\sigma_x\sigma_z - \sigma_z\sigma_x\rangle_\psi$$

is not strong enough to make this prediction since in case of $\Delta = 0$ the lower boundary on the right hand site becomes zero. However, there is an additional term for the lower boundary that usually is dropped but required in the present case in order to give a non-zero lower boundary. The details can be found in Appendix A.

will see, entanglement is closely related to the well-known binding problem of cognitive science (Smolensky, 1990) in this case.

Closing this section we will give a simple illustration of quantum entanglement in a two qubit state. Suppose we have prepared the following entangled pure state describing two objects 1 and 2, which is called the Bell-state:

$$\psi_B = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle). \tag{15}$$

We will demonstrate now that classical probabilities lead to wrong predictions concerning the measurement of (stochastic) *correlations* between certain observables.

Let us assume four binary random variables A_1, A_2, B_1, B_2 with the range $\{-1, +1\}$, as described in classical probability theory. The intention is that A_1 and A_2 represent the stochastic outcome of two different kinds of measurement concerning the first object, and B_1 and B_2 represent the stochastic outcome of two kinds of measurement concerning the second object. Defining correlations between pairs of these random variables in the usual way,

$$C(A_i, B_j) = E_\mu(A_i B_j) \tag{16}$$

we are able to derive the following inequality, which is a version of Bell's inequality (Vedral, 2006).[15]

$$E_\mu(A_1B_1) + E_\mu(A_2B_1) + E_\mu(A_1B_2) - E_\mu(A_2B_2) \leqslant 2 \tag{17}$$

The inequality expresses an upper boundary that restricts the correlations between the random variables A_i and B_j. The same inequality also should apply in the quantum case if quantum theory can be rewritten as a classical theory with hidden variables (an idea famously associated with Albert Einstein). However, it is possible to demonstrate that quantum mechanics makes predictions that violate the 'Bell inequality' in the setup considered in the EPR thought experiment. A simple example is as follows:

$$\begin{aligned} &A_1 : \sigma_z \otimes I, \quad A_2 : \sigma_x \otimes I, \\ &B_1 : -\frac{1}{\sqrt{2}} I \otimes (\sigma_z + \sigma_x), \quad B_2 : -\frac{1}{\sqrt{2}} I \otimes (\sigma_z + \sigma_x) \end{aligned} \tag{18}$$

Using the corresponding Pauli operators, the following correlations can be calculated in the Bell-state ψ_B:

$$\begin{aligned} &\text{a.}\quad E(A_1B_1) = -\frac{1}{\sqrt{2}}\langle \sigma_z \otimes (\sigma_z + \sigma_x)\rangle \psi_B = \frac{1}{\sqrt{2}} \\ &\text{b.}\quad E(A_2B_1) = -\frac{1}{\sqrt{2}}\langle \sigma_x \otimes (\sigma_z + \sigma_x)\rangle \psi_B = \frac{1}{\sqrt{2}} \\ &\text{c.}\quad E(A_1B_2) = \frac{1}{\sqrt{2}}\langle \sigma_z \otimes (\sigma_z - \sigma_x)\rangle \psi_B = \frac{1}{\sqrt{2}} \\ &\text{d.}\quad E(A_2B_2) = \frac{1}{\sqrt{2}}\langle \sigma_x \otimes (\sigma_z - \sigma_x)\rangle \psi_B = -\frac{1}{\sqrt{2}} \end{aligned} \tag{19}$$

Calculating the left hand side of inequality in (18) yields $2\sqrt{2} > 2$. Hence, we have found a clear violation of Bell's inequality in (18), and this demonstrates that quantum theory cannot be replaced by a classical theory with hidden parameters that are stochastically modeled. The importance of this inequality will be demonstrated when the question is raised as to whether cognitive phenomena can be described in terms of classical probabilities or if a more general theory in terms of quantum probabilities is required. The essential point is that when the inequality is violated, it indicates that the underlying state is entangled; that is, the aspects of the state exposed by the different experiments are interdependent The Procrustean bed of Bayesian probability theory cannot be used for calculating stochastic behavior in such cases. The situation is analogous to the attempt to map the surface of our globe to a two-dimensional, flat surface. With a sufficiently precise approximation, this is locally possible only; it never is possible to conserve all spatial distances globally on a two dimensional map (Primas, 2007).

4. Two qubits for C.G. Jung's theory of personality

An interesting argument for applying the formal apparatus of quantum theory to the domain of cognition has to do with the flexibility, instability, and context-dependency of meaningful cognitive entities that manifest themselves as fleeting contents of conscious experience. For example, in the domain of language, words can be flexibly used in a variety of different interpretations (e.g. Blutner, 1998; Blutner, 2004). As the properties of small particles are not absolute and not determined until they are observed, the properties of word tokens are not determined until conscious apprehension. Similarly, impressions, ideas and opinions are conceptual entities with analogous properties and likewise invite an analysis in terms of quantum theory. Recently, Aerts et al. (2008) have applied a quantum analysis to a cognitive setting where individuals' opinions were probed. Three different questions for the opinion poll were considered:

Q1 : Are you in favor of the use of nuclear energy?
Q2 : Do you think it would be a good idea to legalize soft-drugs?
Q3 : Do you think capitalism is better than social-democracy? (20)

Interestingly, in such situations, most people do not have a predetermined opinion. Instead, the opinion is formed to a large extent during the process of questioning in a context-dependent way. That means opinions formed by earlier questions can influence the actual opinion construction. Aerts et al. (2008) assume that in such situations, Bell's inequalities can be violated, and, consequently, the values of the corresponding probabilities cannot be fitted into a classical (Kolmogorovian) probability model. Although Aerts et al. (2008), do not present explicit empirical data to prove the point, their argumentation is still convincing. In a related study, Conte et al. (2008) come to a similar conclusion.

[15] One can also consult the following web site: http://www.quantiki.org/wiki/index.php/Bell's_theorem.

Personality tests can similarly be seen as a cognitive setting where individual opinions are probed. Forced-choice questions such as those presented in the examples (1)–(3) are suitable material for checking the statistical framework and looking for quantum effects. In this section we will show that the phenomenon of entanglement fits very naturally into the framework, and, consequently, we expect violations of Bell's inequalities.

In quantum information science, the qubit proves the simplest possibility to represent forced-choice questions.[16] In the last section we showed how qubits can be modeled using a two-dimensional Hilbert space. In this treatment every question/observable can always be represented as linear combinations of the three Pauli operators. Ignoring phase shifts for the moment, i.e. assuming $\langle\sigma_y\rangle = 0$, we are left with two independent operators, σ_z and σ_x. As explained before, the operator σ_z gives a definite *yes*/*no* answer in case the system is in the base state 0 (1). On the other hand, the operator σ_x gives a definite *yes*/*no* answer in case the system is in the superposed state $0 + 1$ ($0 – 1$). Opponent questions can be formulated by negation (multiplication with –1). Hence $\{\sigma_z, -\sigma_z\}$ and $\{\sigma_x, -\sigma_x\}$ form two independent systems of opponent questions.

In contrast, a classical bit-state could be described by a system consisting of two possible worlds, {0, 1}. In such a system, only one independent system of opponent questions can be used to ask whether the state 0 (or 1) is realized. Of course, using the Cartesian product space, *n*-bit states can be realized. Though correlations between the corresponding questions ('random variables') can be expressed in a classical system with a Kolmogorovian probability measure, the idea of entanglement cannot be expressed in this way.

In the following we will propose a formalization of C.G. Jung's theory of personality using the quantum-theoretical framework. From a methodological point of view it is essential to stress that it first needs a reconstruction of Jung's theory before a careful empirical testing can start. We will demonstrate that our reconstruction provides a proper representation of Jung's basic ideas. Further, at the end of this section we will present some first results of a pilot study demonstrating how aspects of the theory can be tested empirically.

The proposed formalization of Jung's theory is using a four-dimensional Hilbert-space for the representation of two qubits. The first qubit relates to Jung's four psychological functions **T**hinking, **F**eeling, **S**ensing and i**N**tuition, which are represented by two groups of projection operators {**T**, **F**} and {**S**, **N**}. The idea to express the four psychological functions by *one* qubit relates to Jung's insight of conceiving the system of these functions as one quaternity by integrating the four functions to one genuine unit. The idea of how to integrate the four functions by using the Pauli spin operators arises when we compare Jung's image of a cross (Fig. 1) to the Bloch sphere (Fig. 2) representing a full qubit. Assuming a vanishing phase shift Δ as a simplification, we see a complete analogy between these two pictures if we assume the following correspondence:

$$\begin{aligned} &T = \sigma_z, F = -\sigma_z (\text{rational functions}) \\ &S = \sigma_x, N = -\sigma_x (\text{irrational functions}) \end{aligned} \quad (21)$$

This correspondence is the key for expressing Jung's four psychological functions within a single qubit system. The stipulation in (21), obviously, is expressing the idea that the rational functions are opposites from each other. The same applies to the irrational functions. A consequence of (21) is that it gives a simple explanation of the eight basic types that result from the different proportions of the expectation values of the psychological functions (cf. formula (13) in case of the original Pauli-operators). This is illustrated in Fig. 5.

Obviously, exactly eight configurations of psychological functions can be realized corresponding to the eight segments of Fig. 1:

1. F>N>S>T	5. T>S>N>F
2. N>F>T>S	6. S>T>F>N
3. N>T>F>S	7. S>F>T>N
4. T>N>S>F	8. F>S>N>T

Further, this schema satisfies the first restriction we have formulated in connection with C.G Jung's theory: if the superior function is rational/irrational then the secondary function must be irrational/rational, and this alternation is continued along the ranking hierarchy.

In the following, we will assume that the different attitudes of a personality (extraverted vs. introverted) can be expressed by a second qubit. This gives the possibility not only of modeling pure extroverted or pure introverted personalities, but also superpositions of extroverted and introverted types. We assume that a corresponding operator σ_z is available with eigenvectors that represent the two oppo-

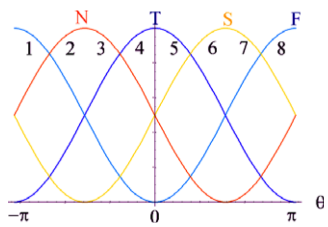

Fig. 5. The 8 types resulting from different proportions of the expectation values for N, T, S, and F.

[16] So far we can see, Atmanspacher, Filk, & Römer (2004) were the first who made use of qubits in Cognitive Science. These authors proposed to understand bistable perception (of the Neckar cube) as a quantum phenomenon. Answering bipolar forced-choice questions is an analogous phenomenon, in our opinion. In both cases the quantum-like behavior of the decision process can be discussed "as a result of the truncation of an extremely complicated system to a two-state system, into which the effect of many uncontrolled variables and influences is lumped in a global way" (Atmanspacher et al., 2004).

nent attitude states extraversion and introversion. Hence, we can write down two observables for extraversion and introversion:

$$E = \sigma_z, \quad I = -\sigma_z(\textbf{E}\text{xtraversion,}\textbf{I}\text{ntroversion}) \tag{22}$$

Though not explicitly discussed in the literature, it also makes sense to introduce an observable that registers states between extraversion and introversion. We will call it M and assume that it gives a definite *yes* answer for an equal superposition of pure introverted and pure extraverted states, i.e.

$$M = \sigma_x(\text{inter}\textbf{M}\text{ediate}) \tag{23}$$

For constructing the full Hilbert space we will make use of the tensor product. Hence, if $|\alpha\rangle$ expresses a certain state of attitude (extraverted, introverted or a superposition of both) and $|\psi\rangle$ a certain psychological state reflecting a certain ranking of the four psychological functions, then $|\alpha\rangle\otimes|\psi\rangle$ expresses a psychological state $|\psi\rangle$ in the attitude $|\alpha\rangle$.

In discussing the type dynamics crucially involved in C.G. Jung's theory (Section 2), we claimed that each person realizes more than one psychological function, and we have stressed the point that opponent psychological functions are realized with contrasting attitudes. In the present formal theory, this idea is expressed by the notion of entanglement. Hence, we will claim that the attitudes are entangled with the psychological functions. Formally, we can write such entangled states $|\Psi\rangle$ in the following way,

$$|\Psi\rangle = |\alpha\rangle \otimes |\psi\rangle - |\alpha\rangle^{\perp} \otimes |\psi\rangle^{\perp} \tag{24}$$

where $\perp$ is an operation that gives the orthogonal state of a certain qubit state. The Bell-state discussed earlier and repeated here

$$\psi_B = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \tag{11}$$

is an instance of the entangled states we have in mind. Using the Bloch sphere and ignoring phase factors, we can write $|\psi\rangle$ and $|\psi\rangle^{\perp}$ in the following way:

$$|\psi\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)|1\rangle, \quad |\psi\rangle^{\perp} = \sin(\theta/2)|0\rangle - \cos(\theta/2)|1\rangle \tag{25}$$

Here $|0\rangle$ and $|1\rangle$ are the eigenvectors of the system of rational functions $\{T, F\}$; the state $|0\rangle$ conforms to the state with a maximal expectation value of T and the state $|1\rangle$ indicates the state with maximal expectation value of F $(= -T)$.

Not unsurprisingly, we can reinterpret Fig. 1 now as a two-dimensional cut of the Bloch sphere showing the possible states $|\psi\rangle$ with a zero phase shift Δ. Obviously, for $\theta = 0$ we get a maximal expectation value of T, for $\theta = \pi/2$ we get a maximal expectation of S, and so on.

In the same way, we can parameterize $|\alpha\rangle$:

$$|\alpha\rangle = \cos(\alpha/2)|0\rangle + \sin(\alpha/2)|1\rangle, \quad |\alpha\rangle^{\perp} = \sin(\alpha/2)|0\rangle - \cos(\alpha/2)|1\rangle \tag{26}$$

In this case, $|0\rangle$ and $|1\rangle$ stand for extraversion and introversion. In Section 3 we saw that quantum entanglement is a phenomenon in which the states of two or more objects are linked together so that one object can no longer be adequately described without fully identifying its counterpart. This interconnection leads to correlations between observable physical properties of systems. In the present case we assume that the psychological functions are entangled with the attitudes we expect. For example, we assume that extraverted thinkers have a dominant **F**eeling-function in the introverted attitude. Similarly, for introverted **S**ensation types, we expect a dominant i**N**tuition-function in the extraverted attitude. As a consequence of this entanglement, we can calculate the following correlations when the operators $\{E, M\}$ for the attitudes and $\{T, S\}$ for the psychological functions are involved:

$$\begin{aligned} &\text{a.}\quad C_\Psi(E,T) = \frac{1}{\sqrt{2}}\langle E\otimes T\rangle_\Psi = \cos(\alpha+\theta) \\ &\text{b.}\quad C_\Psi(E,S) = \frac{1}{\sqrt{2}}\langle E\otimes S\rangle_\Psi = \sin(\alpha+\theta) \\ &\text{c.}\quad C_\Psi(M,T) = \frac{1}{\sqrt{2}}\langle M\otimes T\rangle_\Psi = \sin(\alpha+\theta) \\ &\text{d.}\quad C_\Psi(M,S) = \frac{1}{\sqrt{2}}\langle M\otimes S\rangle_\Psi = -\cos(\alpha+\theta) \end{aligned} \tag{27}$$

For instance, we expect a high correlation $C_\Psi(E, T)$ if the personality is characterized by both a high percentage of extraverted thinking and a high percentage of introverted feeling. If the person is characterized by an entangled state $\Psi(\alpha, \theta)$, then the correlation $C_\Psi(E, T)$ is maximum for $\alpha = \theta = 0$, for instance. Bell's inequality (17) can be applied to the present situation and it says the following:

$$C_\Psi(E,T) + C_\Psi(M,T) + C_\Psi(E,S) - C_\Psi(M,S) \le 2 \tag{28}$$

Inserting the quantum results given in Eq. (27), the inequality becomes

$$2\sin(\alpha+\theta) + 2\cos(\alpha+\theta) \le 2 \tag{29}$$

It is not difficult to see that this inequality is violated in case $0 < \alpha + \theta < \pi/2$. The violation is maximal if $\alpha + \theta = \pi/4$, for example $\alpha = 0$ (pure extraversion) and $\theta = \pi/4$ (state between thinking and sensing, i.e. at the border between regions 5 and 6 of Fig. 1). Calculating the left hand site of inequality (29) gives $2\sqrt{2} > 2$ (we see the close correspondence to the EPR experiment discussed in connection with the Bell state in Eq. (19)). This means that for extraverted thinkers who are strongly supported by the sensing function, we find a maximum violation of Bell's inequality (28). It is obvious that for other types of personalities, other observables must be used in order to measure true violations of the inequality.

Empirical evidence supporting violations of Bell's inequalities proves that quantum mechanics cannot be replaced by a classical theory with hidden variables. Experimentally, it is notoriously difficult to prove violations of one of Bell's inequalities. However, in the physical domain there is now convincing evidence for violations e.g. (Aspect, Dalibard, & Roger, 1982). Though recent attempts to prove Bell's inequality violation in the mental domain were not completely successful (Conte et al., 2008), the situation is not hopeless. It is possible that we will see more success in the present case of personality diagnostics. In this connection, it is important to mention that the experimental situation in cognitive science is potentially different from the situation in particle physics. Particles act in agreement with certain probabilistic laws, but they cannot directly tell us expectation values. For human subjects, however, we can assume that they can give us information directly related to these probabilities. Hence, we expect that human subjects cannot only give yes/no answers; they also can tell us which answer has a higher or a lower probability. Further, they have intuitions about the certainty of a given answer. Theoretically speaking, they have some insight into the underlying quantum probability. And this possibly can simplify the task of empirically finding Bell violations in the domain of personality types.

Next, we have to discuss the second constraint stating that contrasting attitudes have opponent psychological function. This constraint is an immediate consequence of the (type-dynamic) assumption about the entanglement of attitudes and psychological functions as formally described by (24). As a special instance of (24), we take $\alpha = 0$ corresponding to pure states of extraversion/introversion:

$$|\Psi\rangle = |0\rangle \otimes |\psi\rangle - |1\rangle \otimes |\psi\rangle^{\perp} \tag{30}$$

We can now calculate the expectation values for the psychological functions under the two conditions: (i) $E = 1$ and (ii) $I = 1$. We consider region 5 only (cf. Fig. 1), i.e. we take $0 < \theta < \pi/4$. In this region we have $1 > \cos(\theta) > \sin(\theta) > 0$ (corresponding to extraverted thinkers with sensing as auxiliary function):

$$\begin{array}{ll} \langle T/E=1\rangle_\Psi = \cos(\theta) & \langle F/I=1\rangle_\Psi = \cos(\theta) \\ \langle S/E=1\rangle_\Psi = \sin(\theta) & \langle N/I=1\rangle_\Psi = \sin(\theta) \\ \langle N/E=1\rangle_\Psi = -\sin(\theta) & \langle S/I=1\rangle_\Psi = -\sin(\theta) \\ \langle F/E=1\rangle_\Psi = -\cos(\theta) & \langle T/I=1\rangle_\Psi = -\cos(\theta) \end{array} \tag{31}$$

Using these results, we get the ranking 1T 2S 3N 4F for the extraverted attitude, and for the introverted attitude we get the ranking 1F 2N 3S 4T. Since the corresponding functions are opponents ($\{T, F\}$ at rank 1, $\{S, N\}$ at rank 2, etc.), the second restriction of C.G. Jung's theory is satisfied. Obviously, this is a consequence of the entanglement between psychological functions and attitudes.

It is possible to rank the eight attitude-specific psychological functions within one and the same ordinal scale if we make certain stipulations about the relative strength ρ of the two parts of the entangled state. So far we have assumed $\rho = 1$, i.e. that both parts are equally strong. According to the idea of a dynamic process of type elaboration, the process starts without entanglement ($\rho = 0$) and during the process the parameter ρ is slowly increased up to $\rho = 1$ (reflecting the case of an "ideal" personality which is fully integrating its own shadow). Depending on the value of ρ, we have to distinguish two different cases: (a) $\cos(\theta) \cdot \rho < \sin(\theta)$, where the ranking (32a) applies, and (b) $\cos(\theta) \cdot \rho > \sin(\theta)$, where the ranking (32b) applies:

a. 1ET 2ES 3IF 4IN (for $\rho < \tan(\theta)$)
b. 1ET 2IF 3ES 4IN (for $\rho > \tan(\theta)$) (32)

The first configuration (32a) almost agrees with the ordering suggested by C.G. Jung for the first four functions. Only the third and fourth functions are reversed. This configuration applies if the type dynamics has not yet fully developed (and/or the type is close to the boundary between ET and ES). In contrast, the exceptional ranking (32b) applies for fully developed personalities which have integrated their own shadow to a high degree. Of course, it is an empirical question if this possibility can be realized in the type dynamic reality.

Though the present paper takes a theoretical stance, and our primary aim is to provide a plausible transformation of the Jungian ideas in the framework of quantum theory (especially the algebraic theory of Pauli operators), we will suggest some ideas of how the present theory can be tested empirically and how it relates to other reconstructions of Jung's personality theory. We are cautious to consider standard statistical techniques in order to evaluate the present model. Such techniques may be useful in order to evaluate models such as MBTI and to compare it with other conventional models, for example the Big Five (Vacha-Haase & Thompson, 2002). Our doubts about these \statistical techniques have to do with the presumption of a Boolean algebraic foundation, which is clearly wrong in the present case.

First, consider the relation between the MBTI and the present model. As we saw in Section 2, the MBTI assumes four independent bipolar opposites (=Boolean random variables), namely **E**xtraverted/**I**ntroverted, **S**ensing/i**N**tuition), **T**hinking/**F**eeling, and **J**udging/**P**erceiving. Hence, we are left with four parameters in order to fit the full probability distribution of all possible combinations.[17] In contrast, the present model has two free parameters only in the simplest case: the entanglement parameter ρ ($0 \leqslant \rho \leqslant 1$), and the spherical polar coordinate θ ($0 \leqslant \theta < \pi$), provided we stipulate a vanishing phase shift Δ. In the MBTI, the subjects are not asked for the uncertainty of their decisions. However, in the present model this quantity is very significant. A little calculation shows that the standard deviation (as defined by definition (5)) amounts to $\Delta T = \Delta F = |\sin\theta|$ and $\Delta S = \Delta N = |\cos\theta|$ in case of minimal entanglement ($\rho = 1$ or $\rho = 0$), and it is

[17] If we assume dependencies between the random variables and construct a corresponding Bayesian networks to express these dependencies, then we are concerned with up to 15 parameters in order to specify the required probability tables.

always 1 in case of maximal entanglement ($\rho = ½$).[18] For example, pure (extraverted or introverted) thinkers ($\theta = 0$) should be maximally undecided about **S**ensing/ i**N**tuition, and this holds for each degree of entanglement. It further should be noted that the present model predicts the probabilities for **J**udging/**P**erceiving based on the system of formulas (31) and the calculation of the ranking of the psychological functions.

Second, a comparison with the Singer–Loomis approach shows a much closer correspondence to the present model. Both accounts include Jung's claim that the psychological functions cannot be considered in isolation, but always with respect to a definite attitude. Further, both accounts consider a quantitative interpretation of the MBTI scores, though in a different way. Whereas the Singer–Loomis approach uses continuous scores for reflecting decision preferences directly, the present approach asks for the subjective certainty of a decision and constructs the quantitative interpretation from the yes/no decision and the subjective certainty rating of that decision. However, there are also important differences. The Singer–Loomis approach has even more free parameters than the MBTI (corresponding to the fact that the four functions – **S**ensing and i**N**tuition, and **T**hinking and **F**eeling – can act independently of each other), and does not make any predictions about dependencies or correlations of its fundamental parameters. The present model does, as it is shown in formula (27) and (31). In principle, a detailed investigation of parts of the data provided by the Singer–Loomis approach could be used to check the predictions of the present model (considering the only subjects who satisfy the bipolarity assumption). Since our approach can violate Bell's inequality, the standard Bayesian probability theory is violated and cannot account for describing the full distribution of the involved random variables. An interesting task for the future is a careful checking of the empirical data in order to find such violations.

Third, in a first pilot study we have chosen a set of 18 forced-choice questions used in the empirical type test à la Myers-Briggs (see the examples (1)–(3)). 6 questions each conform to the Thinking/Feeling opposition, the **S**ensing/ i**N**tuition opposition, and the **E**xtraverted/ **I**ntroverted opposition. The 18 questions were carefully selected and it was made sure that for each question type Cronbach's alpha[19] was higher then 0.6. The 18 questions were presented to 25 subjects in a random order including 18 filler sentences. The subjects had to answer the questions and they had to say how certain they were about their answers, on a scale from 1 (uncertain) to 5 (very certain). The results were used to calculate judgments for $\langle T\rangle\psi$, $\langle S\rangle\psi$, and $\langle E\rangle\psi$. This was simply done by scaling all answers to the interval [−1, 1]. For example, when an S/N-question was answered with the *S*-alternative (certainty 3) then a judgment for $\langle S\rangle\psi = 3/5$ was taken; when it was answered by the *N*-alternative (certainty 2) then a judgment for $\langle T\rangle\psi = -2/5$ was taken. Finally, a chi-square fit was formed for fitting the 2 model parameters to our two qubit model. For $p = 0.05$ as significance level of rejecting the model hypothesis, 4 of the 25 subjects were ruled out ($\chi^2 > 6.0$). At $p = 0.1$ even 7 of the 25 subjects were ruled out ($\chi^2 > 4.6$). Hence, not all of our subjects conform to the model predictions.

For the subject with the worst χ^2-fit ($\chi^2 = 16.1$) we found $\langle T\rangle\psi = 0.06$ and $\langle S\rangle\psi = 0.03$. As a general provision our model predicts the following condition in case of a zero phase shift Δ:

$$\langle T\rangle_\psi^2 + \langle S\rangle_\psi^2 = 1 \quad (33)$$

In the case under discussion we found $\langle T\rangle_\psi^2 + \langle S\rangle_\psi^2 = .004$, and also in the other 3 cases with a poor model fit the sum $\langle T\rangle_\psi^2 + \langle S\rangle_\psi^2$ was significantly smaller than 1.0 ($p > 5\%$). One obvious possibility to explain the violations of condition (33) is to assume a non-zero phase shift Δ. As it can be seen from Eq. (13), this leads to an extra factor $\cos(\Delta)$ in the *S*/*N* component. For example, assuming a maximum phase shift of $\pi/2$ and an angle $\theta = 0$ provides the parameters for explaining the mentioned case with $\langle T\rangle_\psi^2 + \langle S\rangle_\psi^2 \approx 0$.

Interestingly, the sum $\langle T\rangle_\psi^2 + \langle S\rangle_\psi^2$ was never found significantly higher than 1. This observation supports the general model assuming a phase shift Δ which possibly is non-zero. This model predicts $\langle T\rangle_\psi^2 + \langle S\rangle_\psi^2 \le 1$. It is a significant point that the two qubit model can be used to describe Jung's personality theory by means of some simplifying stipulation ($\Delta = 0$). From a methodological perspective a second aspect is even more important: the theoretical framework can help to overcome possible empirical shortcomings by using the full expressivity of the quantum-theoretical framework – allowing for $\Delta \neq 0$.

Finally, we will make some remarks about cognitive operations. Several authors who consider the framework of quantum theory useful for applications in the cognitive domain (Aerts et al., 2005; Aerts et al., 2008; Busemeyer et al., 2006; Khrennikov, 2003) argue that cognitive operations are modeled best by unitary transformations (i.e. transformations which do not change the scalar product of two involved states). Interestingly, the unitary transformation known as X-gate (Vedral, 2006) realizes the cognitive operation connected to Jung's idea of the shadow. This transformation maps vectors into their orthogonal counterparts. In the field of socionics many other operations are discussed that can be used to define various relations between different types of personalities. However, a careful discussion of the corresponding unitary operations in the

[18] In the general case, we find $\Delta T = \Delta F = \sqrt{1 - (2\alpha^2 - 1)^2 \cos^2\theta}$ and $\Delta S = \Delta N = \sqrt{1 - (2\alpha^2 - 1)^2 \sin^2\theta}$.

[19] Cronbach's alpha is commonly used as a statistical measure of the internal reliability of a psychometric test. It was first named 'alpha' by Cronbach (1951), as he had intended to continue with further instruments. Cronbach's α measures how well a set of variables or items measures a single, unidimensional latent construct: $\alpha = \frac{N \cdot \bar{r}}{1 + (N-1)\bar{r}}$, where N is the number of components (=questions of a certain type) and $\bar{r}$ is the averaged correlation between the N components.

quantum theoretic framework goes beyond this introductory article and must be left for another occasion.

5. Conclusions

The main goal of this article was to demonstrate that quantum theory, as a mathematical construction, provides a natural framework for giving a sound foundation of C.G. Jung's theory of personality. This claim has nothing to do with any speculations about the molecular, biochemical basis of the macro-psychological construction of the personality and its constituents. In this regard we fully agree with Aerts et al. (2008) who state this general methodological point as follows:

> "For clarity, we emphasize that it is the abstracted formalism which is 'borrowed' from quantum theory, not in any way its microphysical ontology of particles and fields. Our approach thus concerns the formal structure of models that are able to describe cognitive entities and processes with contextuality, not the substrate that implements them in the brain." (Aerts et al., 2008, p. 1)

The basic tenet is simply that notions from quantum physics fit better with the conceptual, algebraic and numerical requirements of the cognitive domain than the traditional modeling of concepts in terms of Boolean algebras and the classical probabilities based upon it. Using the quantum framework, we were able to demonstrate that the four psychological functions are in strict correspondence to the Pauli operators σ_x, $-\sigma_x$, σ_z, $-\sigma_z$ of a single qubit state. It is then straightforward to describe the eight basic types of personalities (resulting from the different proportions of the four psychological functions) as different proportions of the expectation values of the relevant Pauli operators (ignoring phase shifts in the underlying qubit states). Further, it was shown that the quantum theoretic notion of entanglement is very useful to express the Jungian idea of type dynamics and his observation that opponent psychological functions are realized in one and the same person with contrasting attitudes.

Although we think that our basic assumptions fit naturally with the Jungian framework, there is enough room for speculations concerning the details that have still to be filled. This concerns not only the preference ordering of the eight attitude-specific psychological functions depending on the type dynamics and the critical entanglement parameter ρ. It also concerns the assignment of a qualification as conscious or unconscious function. Further, the consequences of assuming non-zero phase factors need more empirical research. It is superfluous to say that we are at the very beginning of an empirical verification of the present explication of C.G. Jung's theory. However, the important methodological point is that the present account is much more *restrictive* than related approaches such as those proposed by Myers-Briggs, Singer–Loomis, and the representatives of socionics. It is for this reason that the present model looks a bit more like science than these related approaches.

On the other hand, the present framework is more *general* than the standard statistical framework (as used in MBTI, for instance). The reason is simply that quantum probabilities are more general than standard (Bayesian) probabilities. The future will show whether we need this generalization, and the answer will definitely be positive if a real violation of one of Bell's inequalities can be demonstrated.

Acknowledgments

We thank Stefan Blutner for a careful reading of the manuscript for this paper. Further, we thank many colleagues for hints and valuable criticism including Peter beim Graben, Douwe Oosterhout, Elio Conte, Marten van Someren, and two anonymous referees of Cognitive System Research. A special thank goes to Jason Mattausch for his insightful comments and a careful checking of our English. Of course, the usual clause applies, and we feel responsible for all the remaining errors.

Appendix A. Proving the uncertainty principle

Fick (1968) gives the following general derivation of Heisenberg's uncertainty principle (p. 191ff):

$$\boldsymbol{a} = \mathbf{a} - \langle \mathbf{a} \rangle_\psi \mathbf{1}$$

$$\boldsymbol{b} = \mathbf{b} - \langle \mathbf{b} \rangle_\psi \mathbf{1}$$

$$\Delta^2(\mathbf{a}) = \|\boldsymbol{a}\psi\|^2$$

$$\Delta^2(\mathbf{b}) = \|\boldsymbol{b}\psi\|^2$$

We need Schwarz' inequality: $\|\varphi\| \cdot \|\chi\| \geqslant |\langle \varphi|\chi\rangle|$.

$$\Delta^2(\mathbf{a})\Delta^2(\mathbf{b}) = \|\boldsymbol{a}\psi\|^2\|\boldsymbol{b}\psi\|^2 \geq |\langle \boldsymbol{a}\psi \boldsymbol{b}\psi\rangle|^2 = \langle \boldsymbol{a}\psi|\boldsymbol{b}\psi\rangle\langle \boldsymbol{b}\psi|\boldsymbol{a}\psi\rangle$$
$$= \langle \psi|\boldsymbol{ab}\psi\rangle\langle \psi|\boldsymbol{ba}\psi\rangle = \langle \boldsymbol{ab}\rangle_\psi\langle \boldsymbol{ba}\rangle_\psi$$

Decomposing the operators $\boldsymbol{ab}$ and $\boldsymbol{ba}$ in the two Hermitean operators ½ ($\boldsymbol{ab} + \boldsymbol{ba}$ and $1/2i$ ($\boldsymbol{ab} - \boldsymbol{ba}$) we get

$$\boldsymbol{ab} = (\boldsymbol{ab} + \boldsymbol{ba})/2 + i(\boldsymbol{ab} - \boldsymbol{ba})/2;$$
$$\boldsymbol{ba} = (\boldsymbol{ab} + \boldsymbol{ba})/2i(\boldsymbol{ab} - \boldsymbol{ba})/2i$$

From that the unequality reads

$$\Delta^2(\mathbf{a})\Delta^2(\mathbf{b}) \geq (\langle \boldsymbol{ab} - \boldsymbol{ba}\rangle/2)^2 + (\langle \boldsymbol{ab} - \boldsymbol{ba}\rangle/2i)^2$$

Inserting the definitions for **a** and **b** the following inequality results:

$$(*)\Delta^2(\mathbf{a})\Delta^2(\mathbf{b}) \geq (\langle \mathbf{ab} - \mathbf{ba}\rangle/2 - \langle \mathbf{a}\rangle\langle \mathbf{b}\rangle)^2 + (\langle \mathbf{ab}\rangle - \langle \mathbf{ba}\rangle/2i)^2$$

Erasing the first (non-negative) part, the usual formulation will result:

$$(**)\Delta(\mathbf{a})\Delta(\mathbf{b}) \geq |1/2i < [\mathbf{a}, \mathbf{b}] > |.$$

In case of the Pauli operators σ_x and σ_z and state $|\psi\rangle = \cos(\theta/2)\ |0\rangle + \sin(\theta/2)\ |1\rangle$ the inequality (**) makes a trivial statement only:

$$\Delta\psi(\sigma_x)\Delta\psi(\sigma_z) \geq 0(\text{since}1/2|\langle[\sigma_x, \sigma_z]\rangle_\psi| = |\langle\sigma_y\rangle| = 0)$$

However, if we respect the original inequality (*) then we get the following stronger result: $\Delta\psi(\sigma_x)\ \Delta\psi(\sigma_z) \geqslant |\sin(\theta)\text{-}\cos(\theta)|$ (cf. Fig. 3).

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