Complementarity and Quantum Cognition

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Abstract: The idea of complementarity is one of the key concepts of quantum mechanics. Yet, the idea was originally developed in William James' psychology of consciousness. Recently, it was re-applied to the humanities and forms one of the pillars of modern quantum cognition. I will explain two different concepts of complementarity: Niels Bohr's ontic conception, and Werner Heisenberg's epistemic conception. Furthermore, I will give an independent motivation of the epistemic conception based on the so-called operational interpretation of quantum theory, which has powerfully been applied in the domain of quantum cognition. Finally, I will give examples illustrating the potency of complementarity in the domains of bounded rationality and survey research. Concerning the broad topic of consciousness, I will focus on the psychological aspects of awareness. This closes the circle spanning complementarity, quantum cognition, the operational interpretation, and consciousness.

1. Introduction

The idea of complementarity goes back to William James (1842-1910), a famous psychologist. Niels Bohr applied the idea to physics. Even though he suggested applying the idea also to the social sciences as well, he did not make any important steps in this direction. Consequently, the development of psychological theories based on such insights was delayed by more than 100 years. Only recently the new field of quantum cognition has been developed by researchers such as Diederick Aerts (Aerts, 1982, 2009), Elio Conte (Elio Conte, 1983; Elio Conte et al., 2008), Harald Atmanspacher (Atmanspacher, Römer, & Walach, 2002), Jerome Busemeyer (Busemeyer & Bruza, 2012), Peter beim Graben (beim Graben, 2004), and many others.

Even though the protagonists of quantum cognition have always stressed the point that they do not intend to reduce cognitive psychology to physics in any way and that they only see fruitful applications of the mathematical ideas developed in physics to the domain of psychology, the development of quantum cognition has been hindered by many misunderstandings and confusions.[1]

The aim of this article is to argue that a particular conception of complementarity – Heisenberg's epistemic conception is the key conception for understanding quantum cognition, both from the foundational as from the empirical perspective. This is especially valid if we adopt Görnitz' recent reconstruction of

[1] An almost complete realization of such misunderstandings is accomplished in Hümmler (2017).

Heisenberg's conception (Görnitz, 2011). This view paves the way for a novel view on quantum cognition – one that stresses the explanatory role of abstract qubits and at the same time is a starting point for a novel and scientific solution of the mind-body problem (Görnitz, 2018; Görnitz & Görnitz, 2016; Mann & Mann, 2017).[2]

Here is a short outline of the present paper. In Section 2, I will explain Bohr's and Heisenberg's different ideas of complementarity. Section 3 discusses the role of Heisenberg's epistemic conception of complementarity in the context of modern quantum cognition. I will give some examples illustrating the potency of complementarity in the domains of bounded rationality and survey research. Section 4 is devoted to a foundational issue. It will explain **why** we find non-commuting observables in those domains. We trace the problem to the operational interpretation of measurement (question answering) and the psychological idea of resource limitations in cognitive processing. This immediately relates to the role of consciousness as awareness (Section 5). This closes the circle spanning complementarity, quantum cognition, the operational interpretation, and consciousness.

2. Complementarity

Originally, the idea of complementarity came from the psychology of consciousness, in particular from the writings of William James:

> It must be admitted, therefore, that in certain persons, at least, the total possible consciousness may be split into parts which coexist but mutually ignore each other, and share the objects of knowledge between them. More remarkable still, they are complementary. (James, 1890, p. 206)

As noted by Max Jammer (1989), Niels Bohr was acquainted with the writings of James, and he borrowed that idea from him. Similarly, in a letter to Stapp, Werner Heisenberg mentions "that Niels Bohr was very interested in the ideas of William James". (Stapp 1972, p. 1112).

In turn, Bohr introduced the idea into physics (originally as complementarity of momentum and position), and he proposed its application beyond physics to human knowledge in general. However, his physical conception of complementarity is quite different from James', and his often-cited claim to apply it to human knowledge was put to practice by Bohr. In chapter VII of his book (James 1890) – on more than 10 pages – James describes several phenomena which illustrate the splitting of consciousness into parts that are not accessible from each other. For example, these phenomena concern the

"unconsciousness in hysterics" (p. 202), partial blindness under "post-hypnotic suggestion" (p. 207) or the splitting of a person in several selves in "alcoholic delirium" (p.208). One example describes the common situation of partial anaesthesia:

> The mother who is asleep to every sound but the stirrings of her babe, evidently has the babe-portion of her auditory sensibility systematically awake. Relatively to that, the rest of her mind is in

[2] See also the impressive overview provided by Walach (2019) concerning the topic of health.

> a state of systematized anaesthesia. That department, split off and disconnected from the sleeping part, can none the less wake the latter up in case of need. (p. 213)

Another example refers to the famous subject "Lucie" who was in a state of "post-hypnotic suggestion" and who could – among alol the cards covering her lap – only see those cards that were not a multiple of

1. She was particularly blind to numbers such as 9, 12, 15. Hence, the part consisting of the multiples of 3 was split off and disconnected from the part of numbers. However, under special conditions, when she was not asked to tell which cards she saw but to write them down, the other part of the numbers was accessible (p. 207).

Following Blutner and beim Graben (2016), it seems adequate to use the term 'autoepistemic accessibility' to refer to these phenomena (see also, beim Graben & Atmanspacher, 2006). I will use the term 'autoepistemic' to refer to the epistemic states of human subjects who can reflect on their own epistemic states. If two different states are not simultaneously epistemically accessible to the subject under discussion, then they can be seen as complementary in James' sense.

By contrast, Bohr's concept of complementarity is clearly not a copy of James' epistemic conception.

Bohr's conception refers to the laws of nature rather than to the idea of (auto)epistemic accessibility as in James' writings. In other words, it is an ontic conception rather than an epistemic one.[3]

A concept closely related to Bohr's concept of complementarity is Heisenberg's famous uncertainty principle.[4] In his book, *Die physikalischen Prinzipien der Quantentheorie*, Heisenberg (1944, p. 9) starts his introduction of the uncertainty relation with the idea that all facts of atomic physics that are describable in space and time have to be describable in the wave picture as well. In the simplest case, a particle can be described in the wave picture by a 'wave packet'. However, for a wave packet no precise location and no precise velocity can be defined since the wave packet tends to be dispersed over the whole space. According to the simple laws of optics the following uncertainty relation can be derived

(1) $\Delta q \cdot \Delta p \geq h$

Hereby, Δq and Δp denote the standard deviation (measuring the dispersion) of position and momentum, respectively. The constant h is given as Planck's quantum of action relating radiation energy (E) to frequency (f) in the equation $E = h \cdot f$.

After explaining this optical picture of the uncertainty relation, Heisenberg makes clear that the uncertainty relations can be derived without reference to the wave picture by using the general schemes of quantum theory and its physical interpretation. Generally, it is the complementarity of certain observables (expressed by their non-commutativity or order-dependence of the relevant operators) that allows the derivation of uncertainty relations.

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[3] For more details, the reader is referred to Howard (2004) and to Blutner & beim Graben (2016).

[4] The term *uncertainty principle* is a translation of the German term *Unschärfeprinzip* or *Unbestimmtheitsprinzip*.

The following table gives a schematic illustration of the main differences of Bohr´s and Heisenberg's view of complementarity demonstrating that the so-called Copenhagen interpretation of quantum theory is not as uniform as usually assumed.

Niels Bohr	**Werner Heisenberg**
Following **Kant's** philosophy in assuming that human beings are endowed with the ability to think and imagine according to certain classical categories and schemas	Following **Einstein's** leadership in reinterpreting basic concepts of physics, such as time, position, momentum
Ontic interpretation of complementarity (referring to the laws of nature)	**Epistemic** interpretation of complementarity (referring to autoepistemic accessibility)
No wave packet collapse, no antirealism, no subjectivism	Measurement as wave packet collapse, subjective probabilities

Tab.1 The Copenhagen Interpretation. See Howard (2004) for more details

Let us return now to the interpretational problem for probabilities. In quantum theory, a deep problem concerns the nature of the state vector. Here, we will assume an epistemic interpretation of the state vector. This clearly is the view of Heisenberg's Copenhagen interpretation with a subjective interpretation of probabilities. As made clear recently, this view does not necessarily entail observer-induced wave packet collapse (Barnum et al. 2000; Caves et al. 2002a, 2002b). More importantly, this picture conforms to the predominant picture of the Bayesian interpretation in Artificial Intelligence and Cognitive Psychology. Hence, it is plausible to take this interpretation as the basic conception of probability in quantum cognition (for more details, see Reinhard Blutner & beim Graben, 2016).

In the following section, we will define the notion of (auto)epistemic accessibility based on the operational interpretation of quantum physics. The operational setting suggests a particular algebraic structure for modelling propositions, one that is very different from the classical Boolean setting. The Boolean setting allows to model propositions as sets of possible worlds (with the operation of union, intersection and complement for the basic propositional operations). By contrast, the operational setting motivates a non-Boolean algebraic structure that favours modelling propositions by subspaces of a Hilbert space or by projection operators of the Hilbert space and the corresponding lattice-theoretic operations. The algebra of propositions is defined by non-statistical axioms. Hence, the operational understanding does not require any notion of probability. The concept of probability will emerge by means of a measure function, its subjective interpretation can be motivated by a (quantum) de Finetti representation theorem (Barnum et al. 2000; Caves et al. 2002a, 2002b). Hereby, probabilities are taken to be degrees of belief, which are justified by axioms of fair bedding behaviour.

3. Quantum cognition

Quantum cognition is a research field that applies ideas from quantum physics and quantum information science to develop radically new models of a variety of cognitive phenomena ranging from human memory, information retrieval, and human language to decision making, social interaction, personality psychology, and philosophy of mind.

The initial motivation for this new research field is quite simple and rather unmysterious. It has to do with the assumed algebraic structure of the inner world of ideas, concepts, and propositions. Boole and other great logicians of the 19th century assumed that thinking is like doing regular algebra in following strict rules exhibiting associative, distributive, and commutative properties. These are the same rules we can observe when we consider the construction of sets by using union, intersection, and complementation (Boolean algebra).

However, modern cognitive psychology has challenged this view: natural concepts are based on prototypes. As such, natural concepts are geometrical concepts that best can be represented by convex sets (Gärdenfors, 2000, 2014). In this way, a geometric understanding of the conceptual world was born. Now, it is not clear what exactly is the underlying algebra of convex sets. Obviously, the algebra underlying the operation with convex sets is different from the traditional Boolean algebra. Surprisingly, it comes close to the lattice underlying the closed subspaces of a complex vector space (so-called ortho- modular lattice)

Based on work of the great Hungarian mathematician and philosopher John von Neumann it has become visible that the heart of quantum theory is a new kind of probability theory based on ortho-modular lattices rather than Boolean lattices.[5] This theory is more general than the traditional (Boolean-based) probability theory. Interestingly, this approach seems to be powerful enough to solve some hard puzzles known from standard approaches to rationality, logical thinking, and probabilistic reasoning. This opens new horizons for cognitive modeling and their rational foundation. Both for classical probabilities and for quantum probabilities, the probability function of the events of a sample space W are additive measure functions, i.e.

(2) $P(X \cup Y) = P(X) + P(Y) \; for\, disjoint\, X, Y \subseteq W$

In the classical case, we can derive the law of total probability (Eq. 3) immediately from the Boolean axiom of distributivity.

(3) $P(B) = P(A \cap B) + P(\bar{A} \cap B)$

A simple consequence of this law is that

(4) $P(A \cap B) \leq P(B)$

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[5] A classic observation is that the set of projections is naturally a complete orthomodular lattice.

Obviously, the consequence (4) is valid because probabilities are always non-negative. Unfortunately, this result conflicts with common sense observations. A famous example is 'the conjunction fallacy' found by Tversky and Kahneman (1983). In one of their experiments, subjects are presented with a story such as the following one:

> Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations. (Tversky & Kahneman, 1983, p. 298)

After the presentation of the story the subjects are asked to assess the probabilities of several propositions on a numbered scale. We represent the critical propositions only (together with the averaged judgements of the probabilities):

(5)	(A)	Linda is active in the feminist movement	(0.61)
	(B)	Linda is a bank teller.	(0.38)
	(A & then B)	Linda is a bank teller and is active in the feminist movement	(0.51)

Surprisingly, the probability for a conjunction of A and B is higher than that for the proposition B. Let us examine now how this situation can be handled using quantum probabilities. Fortunately, the example allows for the opportunity to be analysed using the simplest structure possible in quantum theory: a two-dimensional vector space. In this approach, one-dimensional subspaces (represented by unit vectors) realize the propositions A and B (and their orthogonal counterparts $\bar{A}$ and $\bar{B}$). Further, the relevant knowledge about the system (given by the description of Linda) can be represented by a simple vector S, called state vector. Basically, the lengths of the projections of the state vectors onto the vectors representing the events under discussion are assumed to represent the (quantum) probabilities of the events (the so-called Born-rule).

The important comparison concerns the statements "Linda is a bank teller" (B) and "Linda is a feminist and a bank teller" (A & then B). The first statement corresponds with P(B) = 0.38 (see Figure 1). The second statement is a conjunction of two statements. Our basis assumption for handling the conjunction is that first the state vector is projected to state A and second the resulting vector is projected to state B (Lüder's rule). The result of this operation is a vector of length 0.51 (see Figure 1).[6]

Figure 1 illustrates that an ordinary two-dimensional vector space is sufficient to give a resolution of the conjunction puzzle. Complex numbers (as required for spanning a true Hilbert space) are not necessary in the present case. Hence, ordinary projections are sufficient to resolve the conjunction fallacy.

[6] Obviously, this operation does not correspond to the intersection of two vector spaces. To distinguish it from the intersection operation A ∩ B, we rename it by "A & then B".

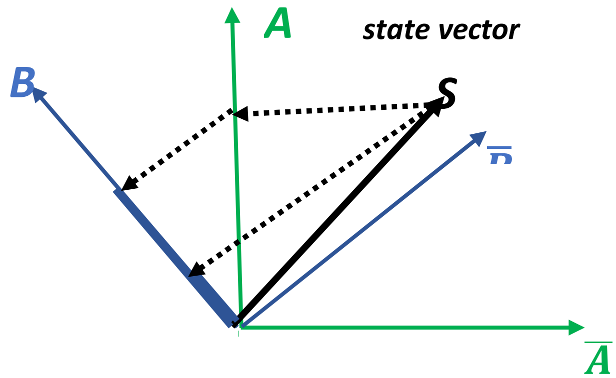

Figure 1: A vector-based explanation of the conjunction fallacy (adapted from Pothos & Busemeyer, 2011)

Another puzzle relates to the disjunction effect. As we will see immediately, this puzzle cannot simply be resolved with a real-valued vector spaces but requires a complex vector space (complex Hilbert space) in order to describe interferences. The disjunction effect occurs when conditioned decisions are considered. Obviously, we can rewrite Eq. (3) in the following form using conditional probabilities:

(6) $P(B) = P(B|A)P(A) + P(B|\bar{A})P(\bar{A})$

The disjunction effect is closely connected to violations of the 'sure-thing principle', one of the basic claims made by a (classically) rational theory of decision making. Let us assume that a decision maker prefers option *B* over option B when knowing that event *A* occurs (i.e., $P(B|A) >$ ½) and likewise when knowing that event *A* does not occur (i.e., $P(B|\bar{A}) >$ ½). Then the 'sure-thing principle' claims that the decision maker should prefer *B* over $\bar{B}$ when not knowing whether *A* occurs or not (i.e., $P(B) >$ ½). If the decision maker refuses *B* (or prefers $\bar{B}$), we have a violation of this principle.

In everyday reasoning, human behaviour is not always consistent with the 'sure thing principle'. For example, Tversky and Kahneman (1983) reported that more students would purchase a non-refundable Hawaiian vacation if they were to know that they had passed or failed an important exam, compared to a situation where the exam outcome was unknown. Specifically, P(*B*|*A*) = 0.54, P(*B*|$\bar{A}$) = 0.57, and P(*B*) = 0.32, where *B* stands for the event of purchasing a Hawaiian vacation, *A* for the event of passing the exam, $\bar{A}$ for the event of not passing the exam, and P for the averaged judgments of probability. Disjunction fallacies are fairly common in behaviour (Busemeyer & Bruza, 2012; Busemeyer, Pothos, Franco, & Trueblood, 2011).

Classical probability theory does not allow patterns such as {P(*B*|*A*) > ½, P(($B|\bar{A}$)) > ½ ,P(*B*) ≤ ½}. Quantum probabilities allow a simple treatment of the puzzle, and a two-dimensional Hilbert space is sufficient for this analysis. As we have seen, in the quantum case, probabilities are calculated from state vectors by a squaring operation. For example, the probability P(B) can be calculated as follows if we introduce the corresponding projection operators $\boldsymbol{A}, \boldsymbol{B}, \bar{\boldsymbol{A}}, \bar{\boldsymbol{B}}$ projecting any state *S* into the subspace indicated by *A*, *B*, $\bar{A}, \bar{B}$:

(7) $P(B) = |\boldsymbol{B}\boldsymbol{A}(S) + \boldsymbol{B}\bar{\boldsymbol{A}}(S)|^2$

Obviously, we have the correspondences

(8) $P(A\ \&\ then\ B) = |BA(S)|^2$; $P(\bar{A}\ \&\ then\ B) = |\boldsymbol{B\bar{A}}(S)|$

With a bit of vector space arithmetic, we can rewrite Eq. (7) as follows (for the technical details, see Blutner & beim Graben, 2016)

(9) $P(B) = P(A\ \&\ then\ B) + P(\bar{A}\ \&\ then\ B) + \sqrt{P(A\ \&\ then\ B)\ \cdot\ P(\bar{A}\ \&\ then\ B)} \cdot cos\Delta$

Hereby, the angle Δ is a phase angle describing a phase shift between the states $\boldsymbol{BA}(S)$ and $\boldsymbol{B\bar{A}}(S)$ making use of a complex vector space.[7]

Considering the numerical values of the Hawaiian vacation example, we get a value of -.23 for the interference term, i.e. the last term of the sum in Equation (9). From this outcome we can, fitting the phase shift parameter: $cos\Delta = -0.42$, i.e. $\Delta = 114°$.

A main topic in applied sociology is the investigation of questions and answers in attitude surveys. Survey researchers have demonstrated repeatedly that the same question often produces quite different answers, depending on the question context (Schuman & Presser, 1981; Sudman & Bradburn, 1982). To cite just one particularly well-documented example, a group of (North-American) subjects were asked whether "the United States should let Communist reporters come in here and send back to their papers the news as they see it?" The other group was asked whether "a Communist country like Russia should let American newspaper reporters come in and send back to their papers the news as they see it?" Support for free access for the Communist reporters varied sharply depending on whether that question preceded or followed the question on American reporters. The differences are quite dramatic: in a study of 1950, 36% accepted communist reporters when the communist question came first and 73% accepted them when the question came second.

Schumann and Presser (1981) described two kinds of ordering effects, which they called 'consistency' and 'contrast' effects. The example with the case of accepted communist reporters illustrates the *consistency effect*, where, in the context of the other question, the answer frequencies are assimilated. In the 'contrast' case, the differences of the answer frequencies are enlarged. In another article, Moore (2002) reports on the identification of two different types of question-order effects termed as 'additive' and 'subtractive'. All four types of question order effects can effectively be handled by single qubits. For a detailed treatment, the reader is referred to Wang and Busemeyer (2013); Wang, Solloway, Shiffrin, and Busemeyer (2014). Interestingly, it is only two parameters that are crucial to define all four order effects: the angle between the two vectors representing the context and the target question and a single phase parameter (for the technical details, see Reinhard Blutner & beim Graben, 2016).

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[7] Note that in a real-valued vector space the states $\boldsymbol{BA}(S)$ and $\boldsymbol{B}\bar{A}(S)$ are both subspaces of B and $cos\Delta$ is either 0 or 1. Hence only by making use of Hilbert spaces the term can vary between -1 and +1.

From an empirical point of view, the framework of quantum states based on a two-dimensional Hilbert space is appropriate to account for an extended series of mental phenomena. Even though many researchers are satisfied with this situation, there are people who have asked for an independent motivation of this vector framework. What are the final reasons for accepting vector spaces, quantum probabilities and the idea of complementarity? And what are the ultimate instruments for bringing together order dependencies and uncertainty relations for handling mental entities? A tentative answer is given in the subsequent section.

4. The Operational Interpretation of Quantum Theory

According to Birkhoff and von Neumann (1936), an operational proposition describing a physical entity (i.e. a, propositions being testable by yes/no experiments) can be represented by an orthogonal projection operator (or by the corresponding closed subspace of the Hilbert space). Hence the Hilbert space stands at the beginning of the theory. This fact poses a deep foundational problem: why using a Hilbert space and not any other geometrical structure?

Interestingly, Mackey (1963) started a distinct project for founding quantum mechanics. He considered the set L of all operational propositions which was restricted in an axiomatic way. In fact, he introduced five axioms on L, and he proved that L is isomorphic to the set of closed subspaces of a generalized Hilbert space. This kind of rational reconstruction of Quantum Mechanics in terms of the actual operational meaning of the fundamental quantum mechanical concepts was further developed by Gleason (1957), the Genova school (Jauch, 1968; Piron, 1976), Foulis and Randall (1972), Solér (1995), and many others.

In these approaches, the set of operational propositions, is structured by an orthomodular lattice instead of an ordinary Boolean algebra.[8] For orthomodular lattices, two main theorems can be proven, which I represent here in an extremely simplified way:

- Piron's theorem (Piron, 1976): Under very general conditions, an orthomodular lattice can be represented by considering the subspaces of a given vector space -- realized by map π.
- Gleason's theorem (Gleason, 1957): Probabilities are the squares of the lengths of the projections of a state vector into a given vector space (or the convex hull of such projections).

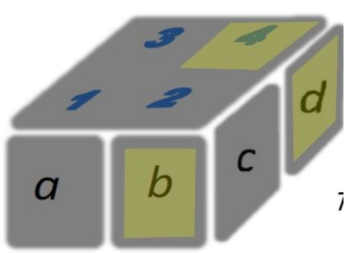
[8] Mathematically, an orthomodular lattice has to satisfy the following axioms (the complement operation is indicated by ', conjunction by $\wedge$, and disjunction by $\vee$): (i) $x'' = x$; (ii) if $x \leq y$ then $y' \leq x'$; (iii) $x \wedge x' = 0$; (iv) if $x \leq y$ then $y = x \vee (x' \wedge y)$ (orthomodular law). The main difference between an orthomodular lattice and a Boolean lattice is that for the latter the law of distributivity is valid but not for the former. Hence, the law of total probability can be derived for Boolean lattices only.

Foulis and colleagues (Foulis, 1999; Foulis & Randall, 1972) give a handy illustration of the basic ideas. It defines the firefly box and its event logic. Assume that there is a *firefly* erratically moving inside the box depicted in Figure 2 (left-hand side). The box has two translucent (but not transparent) windows, one at the front and another one at the right. All other sides of the box are opaque. In principle, the firefly can be situated in one of the four quadrants {1,2,3,4}.[9]

For testing whether the firefly is flashing and where it is, the external observer can take one of two perspectives: (i) looking at the front windows, the flash can be seen on the left-hand side (outcome *a*) or at the right-side (outcome *b*); (ii) looking at the side part, the flash can be seen on the left-hand side (outcome *c*) or on the right-hand side (outcome *d*). Technically, the two perspectives are given by two partitions of the domain *W*={1,2,3,4}: (i) front part = {*a*,*b*}; (ii) side part = {*c*,*d*} (with *a*={1,3}, *b*={2,4}, *c*={1,2}, *d*={3,4}). These two partitions correspond to two Boolean blocks. However, the *union* of these two blocks no longer represents a Boolean lattice. It is weaker and realizes an orthomodular lattice (violating distributivity).

Assume now that the firefly box would have a third window at the top. Then a particular partition would result: top = {{1}, {2}, {3}, {4}}. It relates to an atomic Boolean lattice, the most informative lattice structure that is possible for the given domain *W*. It allows for exactly asking where the firefly is, in segment 1,2,3, or 4. Of course, it is possible to define an operation of 'integration'[10] that would integrate the front perspective with the side perspective into a perspective equivalent to the top perspective.

However, *integration* is a very resource demanding operation quite different from the operation *union* of the lattice structure.[11]



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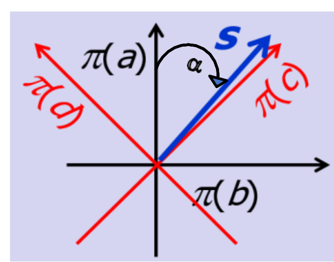


(b)

Figure 2: (a) Illustration of the firefly box. If we look from the front perspective, we can find the firefly in the left part (proposition *a*) or in the right part (proposition *b*). If we look from the side perspective, we can see the firefly in the left part (proposition *c*) or in the right part (proposition *d*). (b) Vector

[9] In the original example, the firefly can be flashing or not (the latter is indicated by being in world 5). We simplify a bit and ignore the world number 5. For a more detailed discussion, cf. Blutner and beim Graben (2016).

[10] This operation is also called 'refinement' and builds a product partition (beim Graben & Atmanspacher, 2006, 2009).

[11] At least, this is true if a theory of resources is assumed as proposed by Halford, Wilson, and Phillips (1998).

representation of the same situation. Hereby, the function $\pi(x)$ assigns vectors to the corresponding propositions (based on Piron's law).

In Figure 2 (b), the state vector *s* is shown. It allows the calculation of concrete probabilities. The angle between the complementary propositions, represented by the vectors $\pi(a)$ and $\pi(c)$ is assumed being

$\pi/4$. In Figure 3, the calculated probabilities are used for presenting the expected mean answers for the verification of the opponent proposition *a* and *b* (yes = +1, no = -1) and for the verification of the complementary propositions *a* and *c*. The parameter α is the angle between the state vector s and the vector $\pi(a)$. Further, the picture shows the standard deviations. In case of the opposite propositions, we

get definite answers at angle $\alpha=\pi/2$. In the case of complementary prosition we do not find an angle with two definite answers for *a* and *c*. As expected from comlementary proposition, a kind of uncertainty principle is valid.

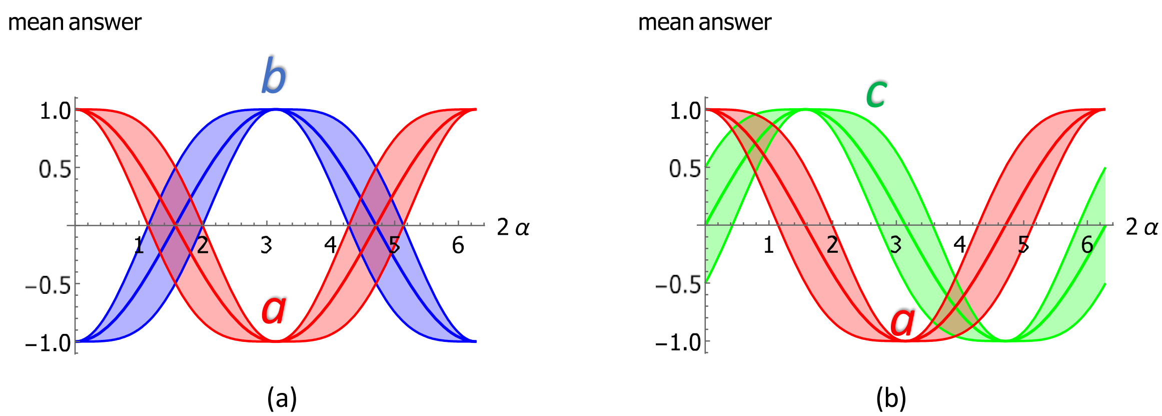
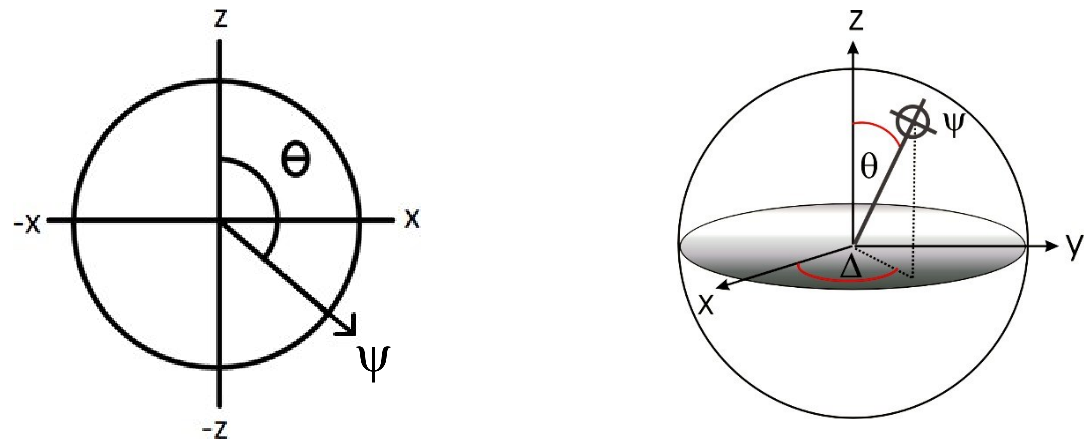


Figure 3: (a) Mean answers for the verification of the opponent proposition *a* and *b* (yes = +1, no = -1); (b) Mean answers for the verification of the complementary propositions *a* and *c*. The parameter α is the angle between the state vector s and the vector $\pi(a)$.

In the present examples, we have illustrated the role of opponent and complementary propositions with real numbers only. However, we should avoid the impression that the introduction of complex numbers is not essential for the treatment of cognitive phenomena. This is visible already in the Hawaiian vacation example and, further, in the mentioned examples of question order effects. Figure 4 shows a parametrization of the state vector ψ in the two-dimensional real vector space and complex vector space. The Bloch circle (left-hand side) characterizes the vector ψ with the azimuthal angle θ; the Bloch sphere (right-hand side) characterizes the vector ψ with the azimuthal angle θ and the phase angle Δ.

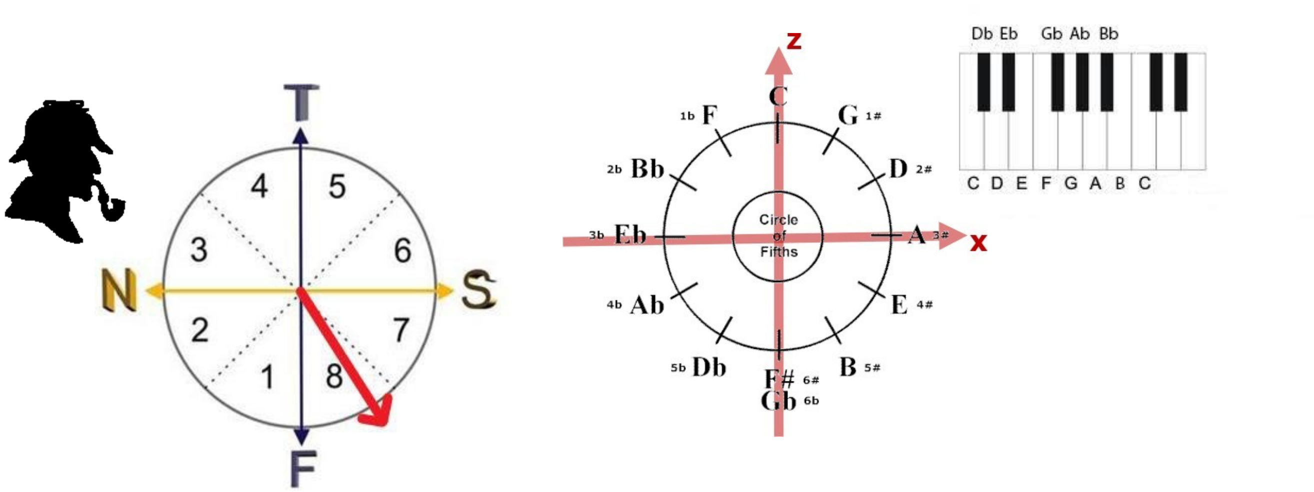
$$\psi = cos\frac{\theta}{2}.\binom{1}{0} + sin\frac{\theta}{2}.\binom{0}{1} \qquad\qquad \psi = cos\frac{\theta}{2}.\binom{1}{0} + sin\frac{\theta}{2}.e^{i\Delta}.\binom{0}{1}$$



(a) (b)

Figure 4: Qubits: (a) in the real vector space (Bloch circle); (b) in the complex vector space (Bloch sphere)

The next illustration (Figure 5) shows two applications of the qubit model.



(a) (b)

Figure 5: Two applications. (a) a qubit realizing C.G. Jung's personality theory; (b) a qubit realization of the circle of fifths in music theory

The left-hand side, Figure 5(a) illustrates the Bloch circle with one axis representing the rational opposites **T**hinking and **F**eeling and a complementary axis representing the irrational opposites (i**N**tuition and **S**ensation) of Jung's personality theory (Jung, 1921). The treatment in terms of qubits was first proposed by Blutner and Hochnadel (2010). The right hand side, Figure 5 (b) illustrates how we can use the Bloch circle for representing the circle of fifths in music theory. As pointed out firstly in Blutner (2017), this way of representation has an empirical impact and provides a new way to represent the attraction potential of chords and scales, which were carefully investigated in cognitive music theory (Krumhansl, 1990).

In both cases, the use of complex vector spaces with phase parameters can consideraby improve the agreement with the available empirical data, as shown by Blutner and Hochnadel (2010) and Blutner (2017). It is not only the observation of complementarity that is essential for quantum theory. Görnitz and Schomäcker (2018) stress the point that the idea of using complex numbers and analytic functions, is essential for quantum theory. It is especially the "requirement of complex differentiability analytic behavior" that ensures the holistic character of quantum theory.[12]

5. Consciousness

A problem for the scientific analysis of consciousness is that there is no unique notion of consciousness but a whole field of different but related notions.

In his Philosophical Investigations, Wittgenstein (1953) introduces the notion of a 'family resemblance' to deal with certain problems of concept formation. Famously, he explained it by the concept family related to the word 'game'. The very same idea can straightforwardly be applied to the term 'consciousness'. Here are some concepts relating to the corresponding family, cf. Chalmers (1996, p. 25 ff):

- Consciousness as attentiveness including the ability to differentiate whether the object of cognition is internal or external. Access Consciousness
- Consciousness as raw experience realizing our Qualia. Phenomenal Consciousness
- Consciousness as mechanism for constructing our Self. Self-Consciousness
- Spiritual Consciousness, connoting the relationship between the mind and God and including the experience of meditation
- Stream of consciousness, altered states of consciousness, animal consciousness, etc. For Chalmers (1996) the family can be divided into two parts:

For now, all that counts is the conceptual distinction between the two notions: what it means for a state to be phenomenal is for it to feel a certain way, and what it means for a state to be psychological is for it

[12] In case of analyzing the phenomenon of 'tonal attraction', a number of different empirical observations can be introduced in terms of a musical gauge field based on the internal symmetry group SU(2) (beim Graben & Blutner, 2019).

to play an appropriate causal role (in explaining behavior, RB). … At a first approximation, phenomenal concepts deal with the first-person aspects of mind, and psychological concepts deal with the third-person aspects. (Chalmers, 1996, pp. 12-16)

Taking the psychological and phenomenal conception of mind into account and adding a material body (physical system), we get the following picture of mind body relations:

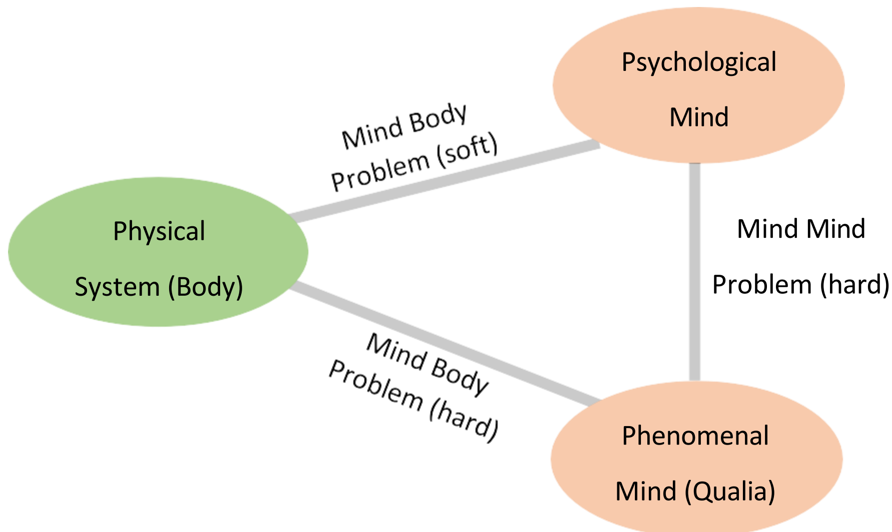


Fig. 6: Mind Body Relations (following Chalmers, 1996)

We consider attentiveness and awareness as related to psychological consciousness. It is the mostly relevant concept for the present discussion. Ignoring the phenomenal mind for the moment, this allows to see the mind body problem as a "soft" problem (in the sense of Chalmers, 1996). Possibly, it can be cracked along the lines pursued by Price and Barrell (2012).[13] [14]

Awareness is a graded conception. We can be aware of something to different degrees. The simplified distinction between automatic and controlled processing (Schneider & Shiffrin, 1977) makes a binary distinction instead of the finer classification by degrees. Awareness is related to cognitive resources.

[13] Also, the notion of spiritual consciousness including the experience of meditation deserves attention. However, even in this case, I see the notion of psychological consciousness in the foreground and the topic of investigation is its correlation with certain physiological parameters. In the sense of Planck (1947), I consider the mind-mind problem and the hard mind-body problem as Schein problems of science. However, this does not have any visible consequences because the "real" scientific problems can be solved based on the psychological conception of mind.

[14] Recently, several models with network-like abilities have been proposed for the modelling of conscious and subconscious processes (Anderson, 1990; Reinhard Blutner, 2004; Grossberg, 2021). In contrast with Görnitz (2018), I cannot see that these ideas give an explication of consciousness in terms of quantum theory. I know a handful of papers only that directly connect neural networks with quantum effects (e.g., Acacio de Barros & Suppes, 2009). These papers, however, do not refer to awareness or consciousness. Hence, the present ideas do not contain a novel (quantum) mechanism for handling consciousness. Rather, they provide some constraints for the route to this goal.

Automatic processing demands less resources than controlled processing. Authors such as Logan (1988) have pointed out that also the allocation of resources is a graded matter.

The foundational approaches to quantum theory discussed in the previous section exploit ideas that do not acknowledge the Hilbert space as a given conceptual framework but rather try to motivate it.

Following the research by Foulis, Randall and colleagues, such a foundational framework is motivated by assuming partial Boolean algebras that describe the perspectives of a cognitive agent. Then there are two possibilities: (i) the perspectives are unified resulting in an orthomodular lattice – a structure that can violate distributivity; (ii) Using Boolean refinement, the perspectives are integrated into a new perspective which is still Boolean. The latter kind of processing demands much more resources than the former. It is at this point that one important foundational aspect enters the theoretical scenery of quantum cognition: resource limitation. Resource limitation has the effect that a cognitive agent cannot simultaneously maintain all possible perspectives and integrate them into a new and more refined structure.

Hence, the whole quantum framework becomes dependent on the psychological concept of awareness and the allocation of (limited) cognitive resources (in the sense of Halford, et al., 1998). If sufficient cognitive resources are not available, there is simply no way to combine the different Boolean blocks in a conjunctive way. Hence, it is not possible to generate a more refined Boolean algebra that allows for rational decisions. Instead, we must accept an orthomodular lattice and the weaker decision structures that are based on quantum probabilities (according to Gleason's theorem).

This argument properly relates to Damasio's theory of consciousness (Damasio, 1994, 1999), in particular to his idea to distinguish a "high-reason" view of decision making from a more emotional but less rational view of decisions. The quantum mode with very fast decisions comes into play if not enough cognitive resources are allocated. By contrast, the more rational classical Boolean mode matters if sufficient resources are available (conscious, controlled processing). In Section 3 we have seen that the idea of qubits forms a powerful instrument for analyzing and solving different puzzles and problems of cognitive psychology. For the last 30 years Thomas Görnitz and colleagues have been exploring if a qubit-based physics is possible (Görnitz, 2011, 2014, 2018; Görnitz & Görnitz, 2016; Görnitz & Schomäcker, 2018). This millennium project is based on ideas of Carl Friedrich von Weizsäcker of seeing matter as information. According to Görnitz, qubits "provide a pre-structure for all entities in natural sciences. They are the basic entities, whereof the physical nature of the brain, on the one hand, and the mental nature of consciousness, on the other hand, were formed during the cosmological and the following biological evolution" (Görnitz, 2018, p. 475). In other words, on the basis of qubits, massless and massive quantum particles can be constructed as well as grammatically formed thoughts (Aerts & Beltran, 2021), as well as musically shaped ideas and mental structures (beim Graben & Blutner, 2019). It goes without saying that handling the physical part of the problem is much more demanding than handling the psychological part.

From a philosophical perspective, the idea that qubits form the base for all quantum structures can help to achieve a deeper understanding of the mind body problem. Qubits are the "substance" for constructing both mental structures as well as physical ones. Hence, mind and body are different aspects of the same elementary basic structures made of information (qubits). Of course, this picture is completely different from Descartes' dualist view of the mind-body problem. Concerning the many different proposals for clarifying the mind-body problem, I think the present idea comes closest to Spinoza's double aspect view and the Indian philosophy in the Vedanta tradition. God is the ultimate for Spinoza, Brahman that for the Hindus and abstract information that for von Weizsäcker, Görnitz, and their followers.[15] Concerning the future development of the idea of complementarity, I think the establishment of a vital connection between the efforts of Thomas Görnitz and mainstream quantum cognition can be beneficial.

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[15] In the field of music, Mannone (2018) refers to another aspect of the mind body problem referring to musical gestures that connect the cognitive-symbolic layer of music to the physical layer of sound.

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