Exercises cont.

3 Bayesian networks

3.1 Independence: Prove the equivalence of the following statements if $\mu(C) \neq 0$**:**

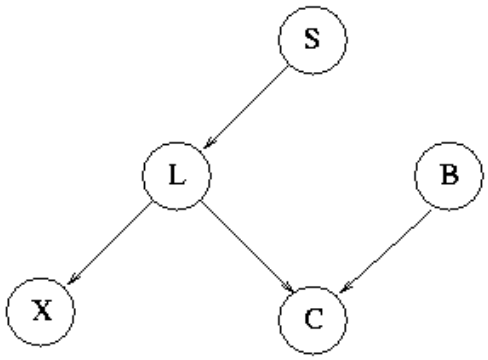
a. $\mu(B \cap C) \neq 0$ implies $\mu(A|B \cap C) = \mu(A|C)$
b. $\mu(A \cap C) \neq 0$ implies $\mu(B|A \cap C) = \mu(B|C)$
c. $\mu(A \cap B|C) = \mu(A|C)\ \mu(B|C)$

3.2 Prove that in a Bayesian network (with a defined Parent-function) the joint probability distribution can be calculated in the following way:

$P(X_1, \ldots, X_n) \quad \prod_{i=1}^{n} P(X_i \mid Parents(X_i)) \quad =$

3.3 One day Apple Jack discovers that his finest apple tree is losing its leaves. Now, he wants to know why this is happening. He knows that if the tree is dry (caused by a drought) there is no mystery - it is very common for trees to lose their leaves during a drought. On the other hand the losing of leaves can be an indication of a disease. How can this situation be modelled by a Bayesian network? Assume that the network consists of three nodes: **Sick**, **Dry**, and **Loses** which can all be in one of two states: Sick can be either "sick" or "not" - Dry can be either "dry" or "not" - and Loses can be either "yes" or "no". The node Sick tells us that the apple tree is sick by being in state "sick". Otherwise, it will be in state "not". The nodes Dry and Loses tell us in the same way if the tree is dry and if the tree is losing its leaves, respectively. Construct intuitively plausible (but still fictive) probability tables!

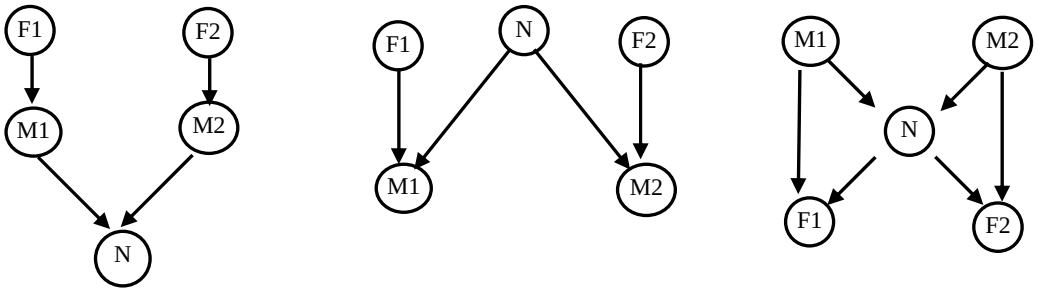
3.4 Consider the following belief network for a medical diagnosis example, where B=Bronchitis, S=Smoker, C=Cough, X=Positive X-ray and L=Lung cancer. Suppose that the prior for a patient being a smoker is 0.25, and the prior for the patient having bronchitis (during winter in Melbourne!) is 0.05.



List the pairs of nodes that are conditionally independent in the following situations (= given evidence):

a. There is no evidence for any of the nodes (absolute independence!).
b. The Lung cancer node is set to true (and there is no other evidence).
c. The Smoker node is set to true (and there is no other evidence).
d. The Cough node is set to true (and there is no other evidence).

3.5 Two astronomers, in different parts of the world, make measurements M_1 and M_2 of the numbers of stars N in some small region of the sky, using their telescopes. Normally, there is a small possibility of error by up to one star. Each telescope can also (with a slightly smaller probability) be badly out of focus (events F_1 and F_2), in which case the scientist will undercount by one star (with a probability of 10%). Consider the three networks shown in the Figure.



(i) (ii) (iii)

a. Which of the belief networks correctly (but not necessarily efficiently) represents the above information?
b. Which is the best network?

3.6 Orville, the robot juggler, drops balls quite often when its battery is low. In previous trials, it has been determined that the probability that it will drop a ball when its battery is low is 0.9. On the other hand when its battery is not low, the probability that it drops a ball is only 0.01. The battery was recharged not so long ago, so there is only a 5% chance that the battery is low. A robot observer, with a somewhat unreliable vision system, reports on whether or not Orville has dropped the ball. This question involves constructing a belief network, containing only Boolean variables, to represent and draw inferences about whether the battery is low depending on how well Orville is juggling.

a. Draw a belief network to represent the problem. Label the network nodes and indicate clearly the direction of the arcs between the nodes.
b. Write down the probability tables showing where the information on how Orville's success is related to the battery level, and the robot observer's accuracy, are encoded in the network.
c. Suppose the robot observer reports that Orville has dropped the ball. What effect does this have on your belief that the battery is low. What type of reasoning is being done?

Hint: Use the nodes BL (**B**attery**L**ow), OD (**O**rville**D**rops), OSD (**O**bserver**S**ees **D**rop)