Exercises cont.

4 Beliefs and defaults

4.1 Is the following pattern of inferences, from {Y>Z, X⇒Y} infer X>Z, an example for inviolable inferences or an example for defeasible inferences? Give examples that fill in this scheme and argue on the basis of these examples!

4.2 Filters: Which of the following sets are Filters? Take $W = \{1,2,3\}$ as set of possible worlds.

a. $F_1 = \{\{1\}, \{2\}, \{1,2\}, \{1,2,3\}\}$
b. $F_2 = \{\{2\}, \{1,2\}, \{1,2,3\}\}$
c. $F_3 = \{\{1,2\}, \{1,2,3\}\}$
d. $F_4 = \{\{2\}, \{1,2\}\}$
e. $F_5 = \{\emptyset, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}\}$

4.3 Intersect two filters. Is the result a filter again? And is the union of two filters a filter? Prove it or give a counterexample.

4.4 Plausibility measure: Prove the following fact:

If Pl(X) is a plausibility function on the algebra $\mathcal{F}$, then the dual Pl'(X) = 1−Pl(¬X) is likewise a plausibility function on the algebra $\mathcal{F}$.

4.5 Prove fact 3, i.e. prove $U \subseteq V$ & $\mathrm{Pl}(U) > \mathrm{Pl}(\neg U) \Rightarrow \mathrm{Pl}(V) > \mathrm{Pl}(\neg V)$

4.6 Take that an agent believes U iff $\mu(U) > ½$. Show that this definition of beliefs does not satisfy closure under conjunction in the general case.

4.7 Take a probability function μ and consider the condition Pl4:
Pl4. If U_0, U_1, and U_2 are pairwise disjoints sets, then $\mu(U_0 \cup U_1) > \mu(U_2)$ & $\mu(U_0 \cup U_2) > \mu(U_1) \Rightarrow \mu(U_0) > \mu(U_1 \cup U_2)$
Give a concrete example (chose a function μ) that shows that Pl4 can be satisfied.
Give also an example that shows that not all probability functions satisfy Pl4.

4.8 Using the rules and axioms of P, prove that

a. $\{(A \wedge B)>C, (A \wedge \neg B)>C\} \mid\!-_{\mathbf{P}} A>C$
b. $\emptyset \mid\!-_{\mathbf{P}} (A \wedge B)>(A \vee B)$
c. $\{A \Rightarrow B, (A \wedge B)>C\} \mid\!-_{\mathbf{P}} A>C$
(remark: X⇒Y means that X→Y is a propositional tautology)

4.9 Prove that the definition (ε-CP) $M \models \phi > \psi$ iff $\mu(\,[\![\psi]\!] \mid [\![\phi]\!]\,) > 1-\varepsilon$ satisfies the conditions LLE, RW, REF of the axiom system **P**. Construct an example (using an arbitrary ε) that shows that AND is not necessarily satisfied.