Exercises cont.

5 Multi-agent systems

5.1 Show that a binary relation is reflexive, symmetric, and transitive if and only if it is reflexive, Euclidian, and transitive

5.2 Take ***K*** as a binary relation and show the following

a. ***K*** is reflexive iff $u \in \underline{\boldsymbol{K}}(u)$ for all worlds u

b. ***K*** is transitive iff $v \in \underline{\boldsymbol{K}}(u) \Rightarrow \underline{\boldsymbol{K}}(v) \subseteq \underline{\boldsymbol{K}}(u)$ for all worlds u, v

c. ***K*** is Euclidian iff $v \in \underline{\boldsymbol{K}}(u) \Rightarrow \underline{\boldsymbol{K}}(v) \supseteq \underline{\boldsymbol{K}}(u)$ for all worlds u, v

5.3 Assume K is reflexive. Show the following:
$(W, \underline{\boldsymbol{K}}, w) \models K\phi$ implicates $(W, \underline{\boldsymbol{K}}, w) \models \phi$ for all $w \in W$

5.4 Assume the following clause: $(W, \underline{\boldsymbol{K}}, w) \models \phi$ implicates $(W, \underline{\boldsymbol{K}}, w) \models K\phi$ for all $w \in W$. Formulate a condition for the relation K such that this clause becomes true. Do the same for the clause $(W, \underline{\boldsymbol{K}}, w) \models \phi$ implicates $(W, \underline{\boldsymbol{K}}, w) \models P\phi$ for all $w \in W$.

5.5 Suppose that a deck consists of three cards labelled A, B, and C. Agent 1 and 2 each get one of these cards; the third card is left face down. Agent 1 saw which card is left face down, but agent 2 did not. Hence, the epistemic situation is different from the situation described in the lecture. Give a graphical representation of the present epistemic situation!

5.6 Prove the validity of the rule of knowledge generalization:
if $M \models \phi$ then $M \models K_i\phi$

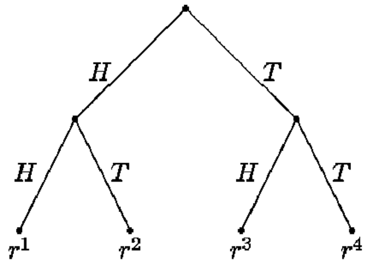
5.7 Assume frames M with a reflexive accessibility relation $\underline{\boldsymbol{K}}_i$. Prove the validity of the *knowledge axiom*: $M \models (K_i\phi \rightarrow \phi)$

5.8 Assume frames M with a transitive accessibility relation $\underline{\boldsymbol{K}}_i$. Prove the validity of the *positive introspection axiom*: $M \models (K_i\phi \rightarrow K_i K_i \phi)$

5.9 Assume frames M with a Euclidian accessibility relation $\underline{\boldsymbol{K}}_i$. Prove the validity of the *negative introspection axiom*: $M \models (P_i\phi \rightarrow K_i P_i \phi)$

5.10 Suppose Alice tosses two coins and sees how the coins land. Although Alice knows the outcome of the first coin toss after she has tossed it, she forgets it after she tosses the second coin. Bob learns how the first coin landed after the second coin is tossed, but does not learn the outcome of the second coin toss. You can model this situation by assuming

- One initial state: ([.], [.], [.])
- Two time-1 states of the form ([X], [X], [*tick*]), where X∈{H,T}
- Four time-2 states of the form ([X_1, X_2], [*tick*, X_2], [X_1, *tick*]), where X_i∈{H,T}



(a) Determine the equivalence classes $\underline{\boldsymbol{K}}_A(r_i, t)$ and $\underline{\boldsymbol{K}}_B(r_i, t)$ for t = 0, 1, 2

(b) Assume uniform runs (equal probabilities) and calculate the probabilities $\mu_{(r1, 1), A}(r_i)$ and $\mu_{(r1, 2), A}(r_i)$

5.11 Analyze the three prisoners puzzle with the help of protocols. Consider both a probabilistic protocol (assuming that the guard is equally likely to say ‘B/C will be executed’ if both options are open according to the rules system) and a deterministic protocol (assuming that the guard makes a deterministic decision in the case where two rules are available). How can you model a situation where the guard’s strategy is unknown to the agent?

5.12 Analyze the Monty Hall Puzzle with the help of protocols. Consider both a probabilistic protocol (assuming that Monty is equally likely to open either of the two doors if two are available according to the rules system) and a deterministic protocol (assuming that Monty makes a deterministic decision in the case where two rules are available, perhaps by preferring the door with the lower number, i.e. he takes door 1 if {1,2} are available and door 2 if {2,3] are available. How can you model a situation where Monty’s strategy is unknown to the agent?