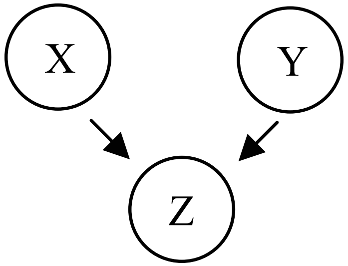
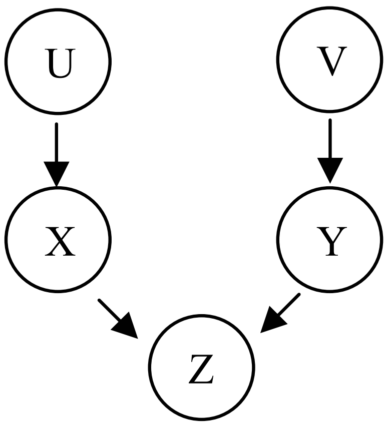
Conditional independence and D-separation

Local semantics: Each node is conditionally independent of its non-descendants given its parents



The local semantics provides a sufficient condition for independence.
In the first example, X and Y can be shown to be independent according to the local semantics.



In the second example, X and Y cannot be shown to be independent, but they still are independent of each other!

$\mu(x,y) = \Sigma_{u,v}\ \mu(x,y,u,v) = \Sigma_{u,v}\ \mu(u)\ \mu(v)\ \mu(x|u)\ \mu(y|v) =$
$\Sigma_{u,v}\ \mu(x)\ \mu(y)\ \mu(u|x)\ \mu(v|y) =$
$\mu(x)\ \mu(y)\ \Sigma_{u,v}\ \mu(u|x)\ \mu(v|y) = \mu(x)\ \mu(y)$

[note: write $\mu(x)$ for $\mu(X=x)$ …]

D-separation provides a much stronger criterion for independence than the local semantics.

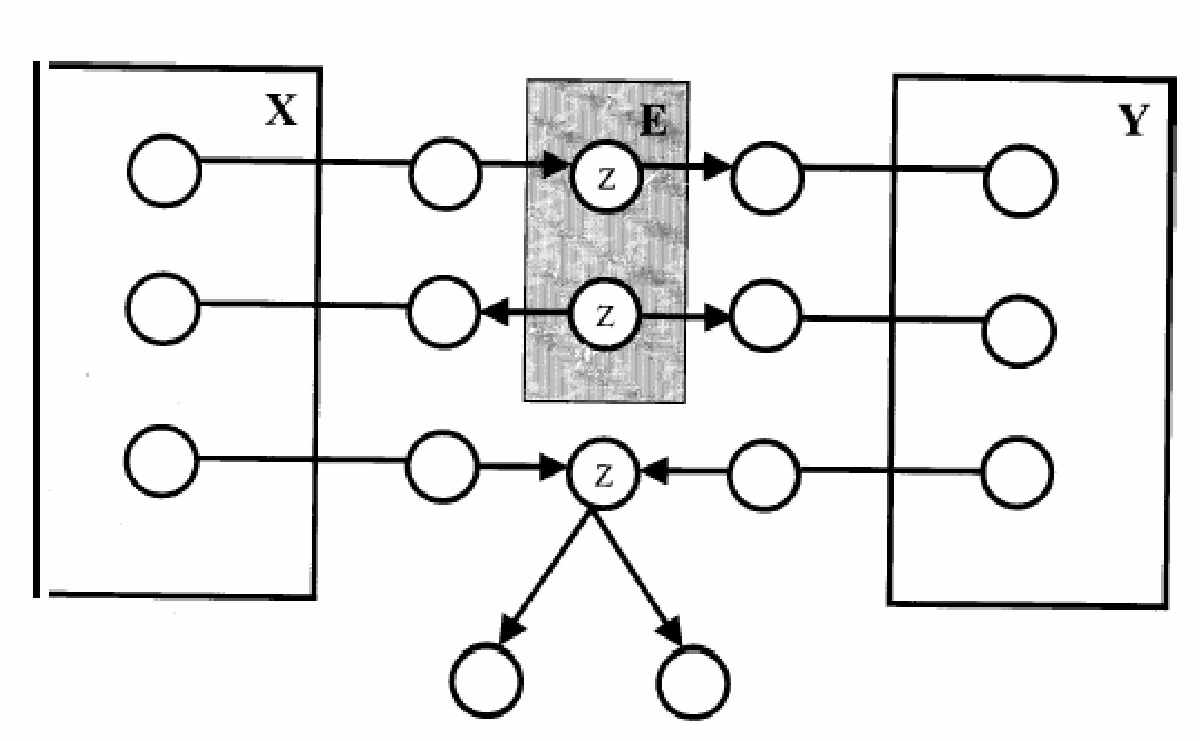
How can we determine whether a set of nodes **X** is independent of another **set** Y, given a set of evidence nodes E?

- If every undirected path from a node in **X** to a node in **Y** is d-separated by **E**, then **X** and **Y** are *conditionallY* independent given E.

- A set of nodes **E** d-separates two sets of nodes **X** and **Y** if every undirected path from a node in **X** to a node in **Y** is blocked given **E**.

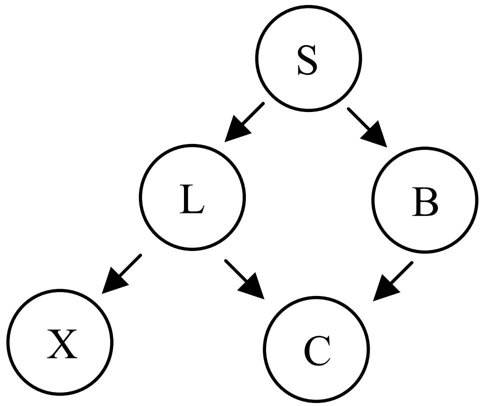
- A path is blocked given a set of nodes E if there is a node Z on the path for which one of three conditions holds:
  1. *Z* is in E and Z has one arrow on the path leading in and one arrow out (chain).
  2. *Z* is in E and Z has both path arrows leading out (common cause).
  3. Neither Z nor any descendant of Z is in E, and both path arrows lead in to Z (common effect)

Three ways in which a path from **X** to **Y** can be blocked, given the evidence **E**. If every path from **X** to **Y** is blocked, then we say that **E** d-separates **X** and **Y** .



E d-separates **X** and **Y**

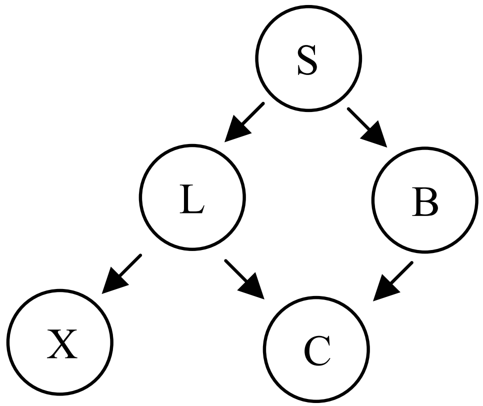
S Smoker
L Lung cancer
B Bronchites
X Positive X ray
C Caugh



Given evidence E, which node pairs are conditionally independent?

1. E = ∅:
2. E = {S}:
3. E = {L}:
4. E = {L, B}:

S Smoker
L Lung cancer
B Bronchites
X Positive X ray
C Caugh



Given evidence E, which node pairs are conditionally independent?

1. E = ∅: None
2. E = {S}: (L, B), (B, X)
3. E = {L}: (X, S), (X, B), (X, C)
4. E = {L, B}: (C, S), (C, X), (X, S)

In the present case, local semantics and D-separation give the same pairs of conditional independence. Generally, D-separation can be stronger (giving more pairs). See homework 3.4!

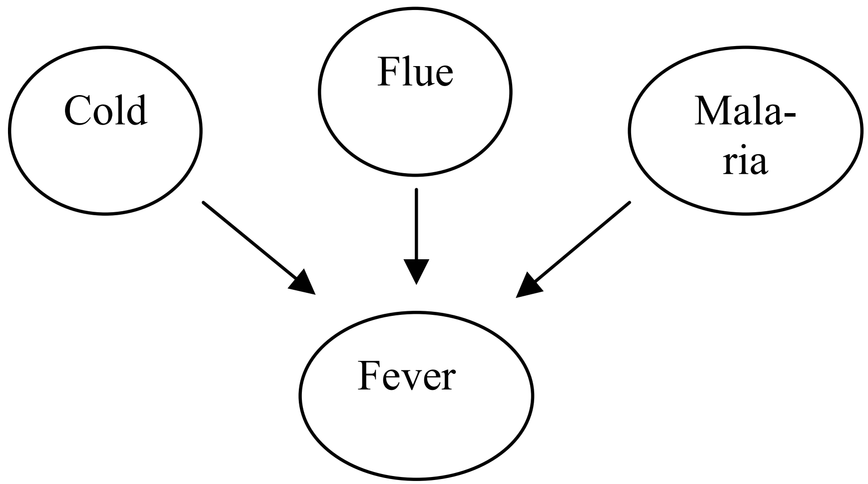
Noisy OR

- The number N of independent entries in the CPT (conditional probability table) grows exponentially with the number of parents (with binary units: $N \sim 2^n - 1$)

- Two ways of overcoming this worst-case scenario:

  – The relation between parents and children is restricted in the sense that there are conditional independencies between the nodes. For instance, if each node has not more than three parents, then $N < 8\,n$

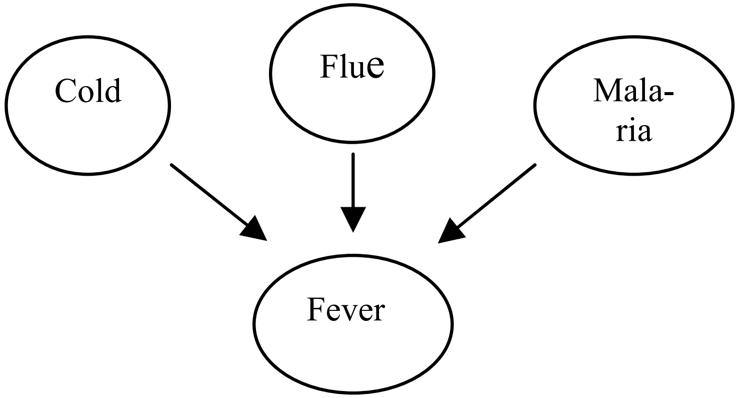
  – Instead of free distributions, often canonical (parameterized) distributions are suggested. The noisy OR is the most popular distribution in the discrete case.



The noisy OR is a generalization of the logical OR. Three assumptions:

1. All possible causes U_i for a event X are listed (you can add a *leak* node)
2. Negated causes $\neg U_i$ do not have any influence on X
3. Independent failure probability q_i for each cause alone.

$$\mu(X|U_1 \ldots U_j, \neg U_{j+1} \ldots \neg U_k) = 1 - \Pi^{j}_{i=1} q_i$$



$\mu(X|U_1 \ldots U_j, \neg U_{j+1} \ldots \neg U_k) = 1 - \Pi^{j}_{i=1} q_i$

Cold	Flu	Malaria	μ(Fever)	μ(¬Fever)
F	F	F	0	1
F	F	**T**	0.9	**0.1**
F	**T**	F	0.8	**0.2**
F	T	T	0.98	0.02 = 0.2 x 0.1
T	F	F	0.4	**0.6**
T	F	T	0.94	0.06 = 0.6 x 0.1
T	T	F	0.88	0.12 = 0.6 x 0.2
T	T	T	0.988	0.012 = 0.6 x 0.2 x 0.1

e.g. μ(¬Fever|Flue&Malaria&¬Cold) = μ(¬Fever|Flue) μ(¬Fever|Malaria) = 0.2 x 0.1

Assume a noisy OR-gate model for $\mu(A|\ E, B)$. Calculate the probability table assuming $\mu(A|\ E, \neg B) = 0.2$ and $\mu(A|\ \neg E, B) = 0.9$