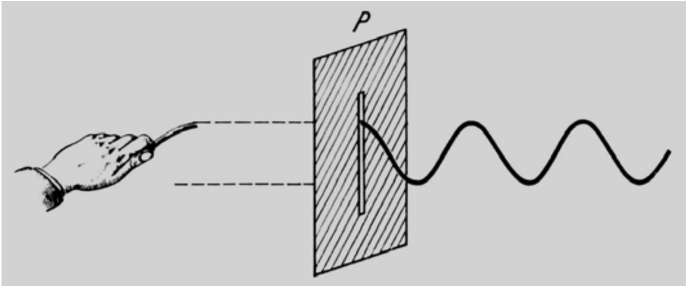
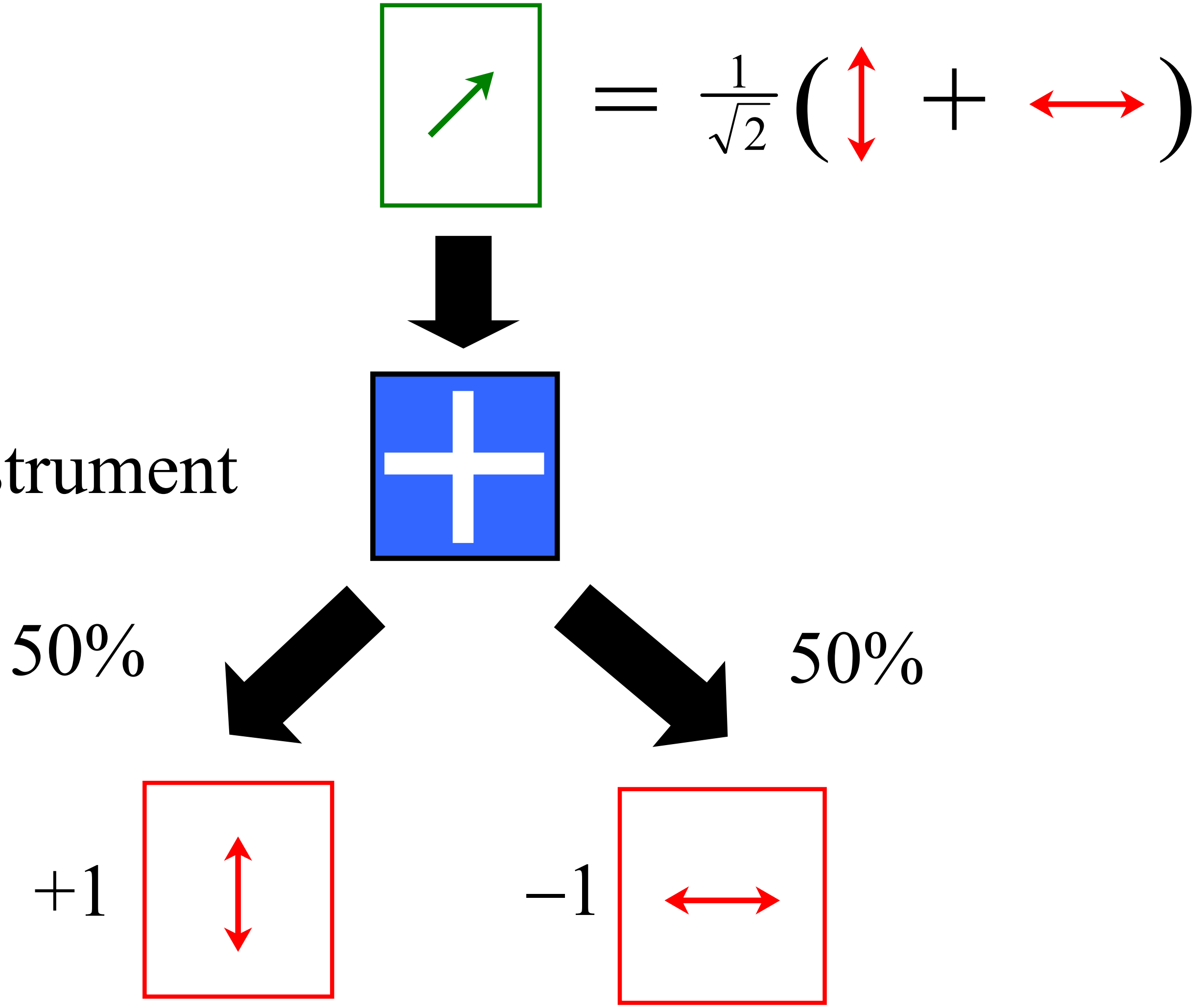
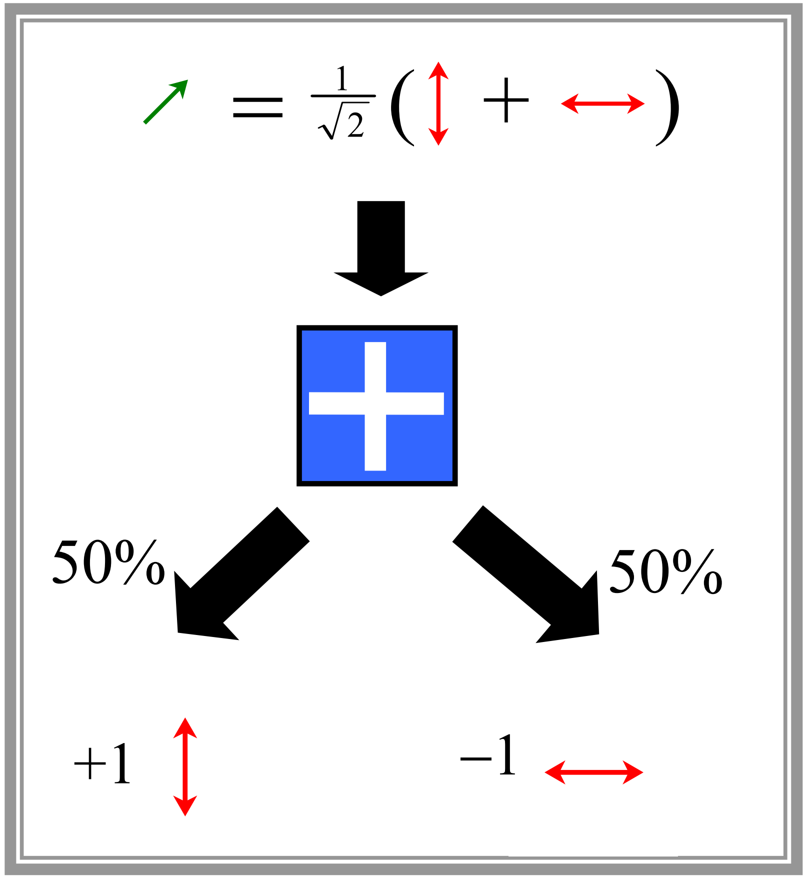


4 Quantum Probabilities

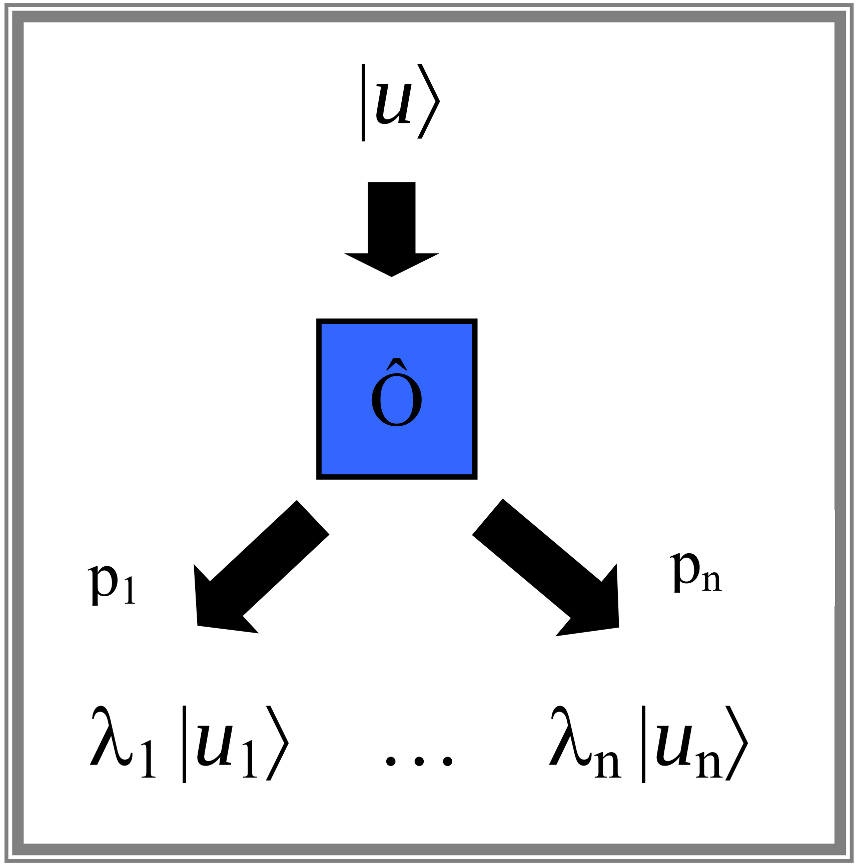
Measuring instrument



The collapse of a pure state in a measurement



A measurement of observable Ô transforms a pure state $|u\rangle\langle u|$ in a mixed state $\Sigma_i\, p_i\, |u_i\rangle\langle u_i|$, where the u_i are the eigenstates of operator Ô and $p_i = |\langle u_i\, , u\rangle|^2$.



Example

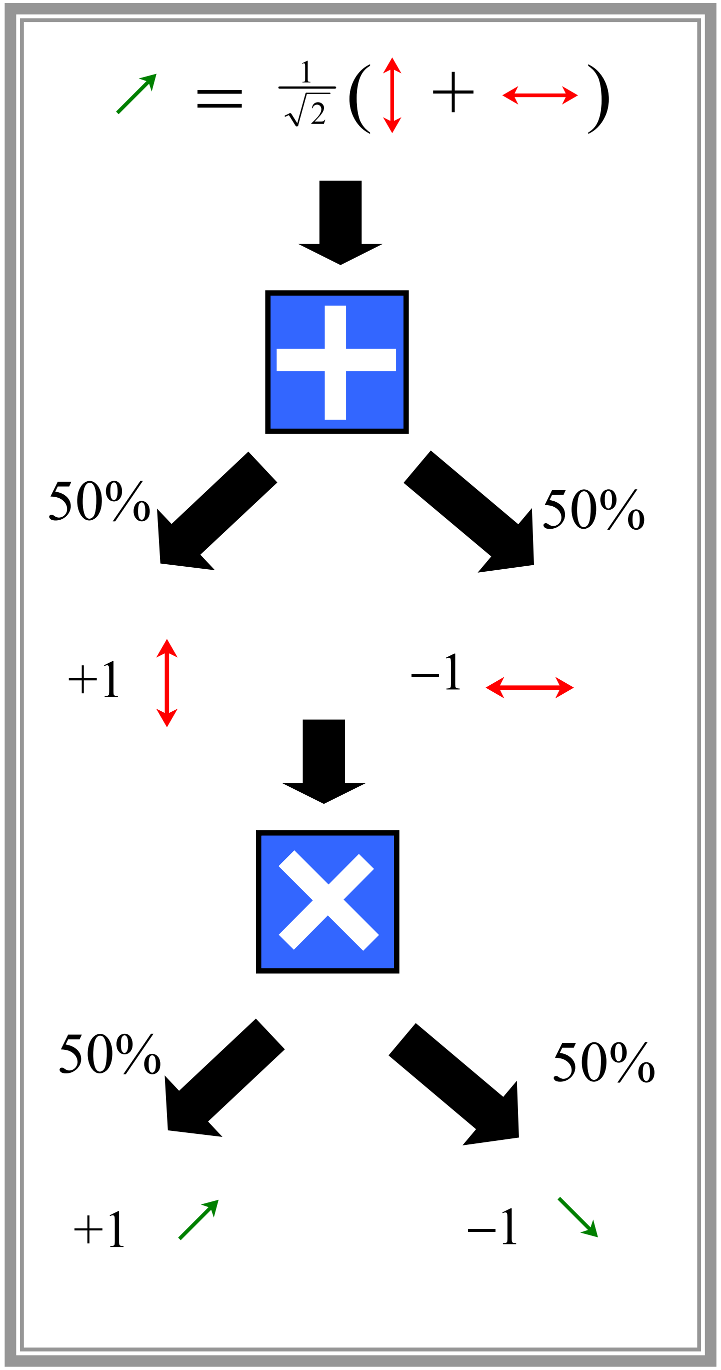
 is described by $\hat{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ with eigenvectors $|\updownarrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|\leftrightarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Now consider

$$|\nearrow\rangle = \tfrac{1}{\sqrt{2}}(|\updownarrow\rangle + |\leftrightarrow\rangle) = \tfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The probability that $\nearrow$ collapses into the eigenstates $\updownarrow$ and $\leftrightarrow$, respectively, is given by the square of the scalar product

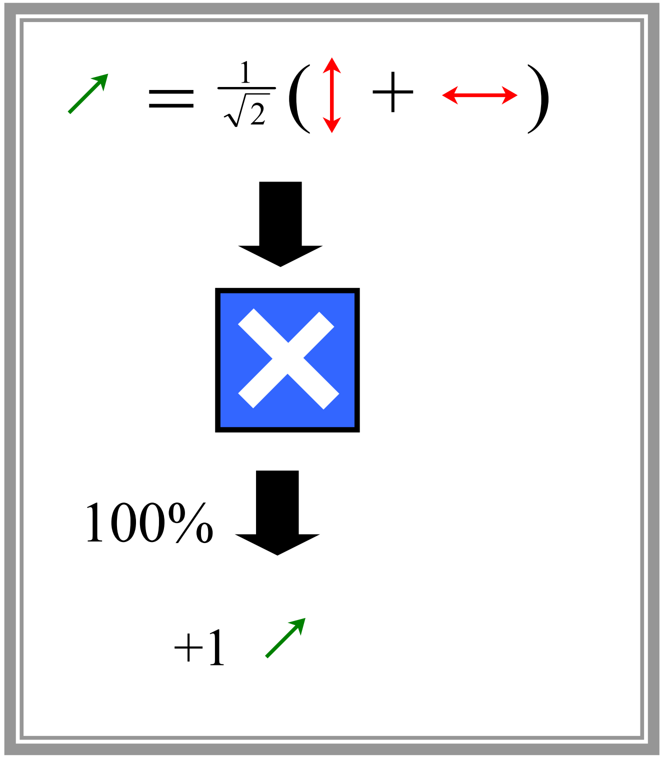
$$p_1 = |\langle \updownarrow, \nearrow \rangle|^2 = ½$$

$$p_2 = |\langle \leftrightarrow, \nearrow \rangle|^2 = ½$$



A puzzle of measurement

A measurement changes the measured state. An *observable* that is *sharp* in a certain state can turn into a *blunt* one by a previous measurement with another instrument!



Definition

- for a pure state u the corresponding *density matrix* is

$$\rho = |u\rangle\langle u|$$

- for a mixed state describing a quantum system that is in the states $u_1, u_2, \ldots, u_n$ with probabilities $p_1, p_2, \ldots, p_n$ ($\Sigma_i\, p_i = 1$) the corresponding *density matrix* is

$$\rho = \Sigma_i\, p_i\, |u_i\rangle\langle u_i|$$ is a *density matrix*

- If a density operator ρ can be written in the form $\rho = |u\rangle\langle u|$ it is said to represent a pure ensemble. Otherwise, it is said to represent a mixed ensemble.

Facts

- A density operator represents a pure ensemble if and only if $\rho^2 = \rho$, or equivalently, if and only if $\mathbf{Tr}(\rho^2) = 1$.

  note: $\mathbf{Tr}(\hat{O}) = \sum_i \langle u_i | \hat{O} u_i \rangle$, for orthonormal basis $\{u_i\}$

- For all ensembles, both pure and mixed,

$$\mathbf{Tr}(\rho^2) \leq 1.$$

- If $\rho = |u\rangle\langle u|$ then $\mathbf{Tr}(\rho\hat{O}) = \langle u|\hat{O}u\rangle$; the expectation value for the observable $\hat{O}$ in $\mathcal{H}$.

- If $\rho = \sum_i p_i \, |u_i\rangle\langle u_i|$, then $\mathbf{Tr}(\rho\hat{O}) = \sum_i p_i \cdot \langle u_i|\hat{O}u_i\rangle$

Definition

Let $\Pi(\mathcal{H})$ be the orthoalgebra of projections on $\mathcal{H}$. A (countably additive) *probability measure* on $\Pi(\mathcal{H})$ is a mapping $\mu : \Pi(\mathcal{H}) \to [0,1]$ such that $\mu(\mathbf{1}) = 1$ and, for any sequence of pair-wise orthogonal projections $\hat{O}_i$, $i = 1,2,\ldots$

$$\mu(\cup_i \hat{O}_i) = \sum_i \mu(\hat{O}_i)$$

Examples

- $\mu_u(\hat{O}) = < u \mid \hat{O}u >$, for a unit vector u of $\mathcal{H}$
- $\mu_\rho(\hat{O}) = \mathbf{Tr}(\rho\hat{O})$, for a density matrix $\rho = \Sigma_i \, p_i \, |u_i\rangle\langle u_i|$

Suppose we are given an arbitrary but consistent description of probabilities for an orthoalgebra of projections and call it a "putative" state. Is there a representation of states that has room for every such putative state. Gleason's famous theorem answers this question affirmatively: take a density operator.

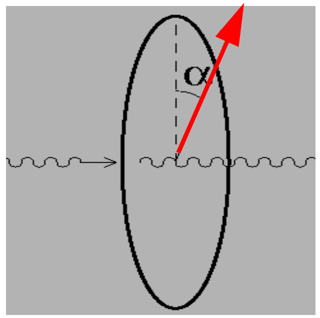
Gleason's Theorem

Let $\mathcal{H}$ have dimension > 2. Then every countably additive probability measure on $\Pi(\mathcal{H})$ has the form $\mu(\hat{O}) = \mathbf{Tr}(\rho\hat{O})$, for a density operator ρ on $\mathcal{H}$.

Example

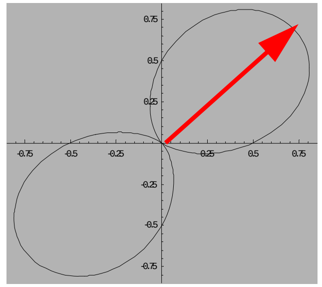
Consider the following projection operator (angel α polarizer):

$$\hat{P}_\alpha = \begin{pmatrix} \cos^2\alpha & \cos\alpha\cdot\sin\alpha \\ \cos\alpha\cdot\sin\alpha & \sin^2\alpha \end{pmatrix}$$



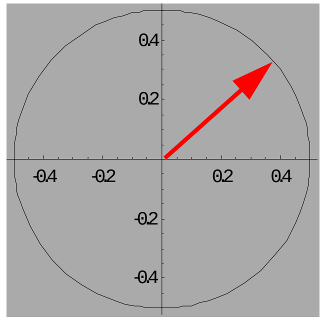
- Distribution induced by $|\nearrow\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$:

  $\mu_{\nearrow}(\hat{P}_\alpha) = \cos\alpha\cdot\sin\alpha + ½ = ½\,(1+\sin(2\alpha))$

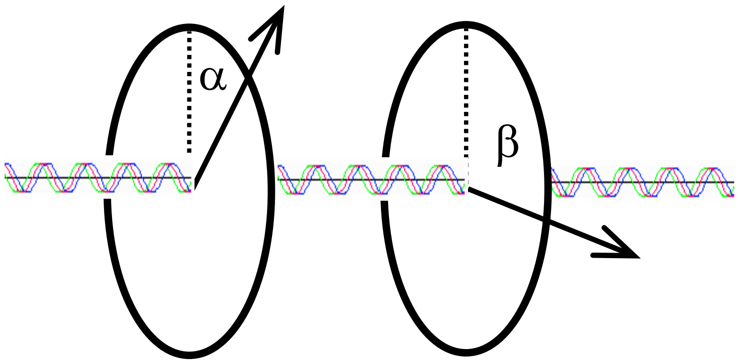


- Distribut. induced by $\{50\%\,|\updownarrow\rangle,\ 50\%\,|\leftrightarrow\rangle\}$:

  $\mu_{\{½\updownarrow,\ ½\leftrightarrow\}}(\hat{P}_\alpha) = ½$



Correlations*



$\mu_\rho(\hat{P}_\alpha \hat{P}_\beta)$ can be taken as describing the correlation between the two polarization filters in the directions α and β. It holds:

$$\mu_\rho(\hat{P}_\alpha \hat{P}_\beta) = \tfrac{1}{2}\cos^2(\alpha-\beta), \text{ with } \rho = \tfrac{1}{2}\,\mathbf{1}$$

Proof

$$\mu_\rho(\hat{P}_\alpha \hat{P}_\beta) = \mathbf{Tr}(\rho\,\hat{P}_\alpha \hat{P}_\beta) =$$

$$\tfrac{1}{2}\,\mathbf{Tr}\left(\begin{pmatrix} \cos^2\alpha & \cos\alpha\cdot\sin\alpha \\ \cos\alpha\cdot\sin\alpha & \sin^2\alpha \end{pmatrix}\begin{pmatrix} \cos^2\beta & \cos\beta\cdot\sin\beta \\ \cos\beta\cdot\sin\beta & \sin^2\beta \end{pmatrix}\right)=$$

$$\tfrac{1}{2}(\cos\alpha\cos\beta + \sin\alpha\sin\beta)^2 = \cos^2(\alpha-\beta)$$

* Kümmerer & Maassen, *Elements of Quantum Probability*

Proposition

For any three Boolean (0-1) random variables P_1, P_2, P_3 on a classical probability space (W, μ) the following inequality holds:

$$\mu(P_1 = 1, P_3 = 0) \leq \mu(P_1 = 1, P_2 = 0) + \mu(P_2 = 1, P_3 = 0)$$

Proof

$$\mu(P_1 = 1, P_3 = 0) =$$
$$\mu(P_1 = 1, \; P_2 = 0, P_3 = 0) + \mu(P_1 = 1, \; P_2 = 1, P_3 = 0) \leq$$
$$\mu(P_1 = 1, P_2 = 0) + \mu(P_2 = 1, P_3 = 0).$$

Does god play dice?

Einstein in a letter to Niels Bohr: "I cannot believe that god plays dice with the cosmos."

Take three polarization filters (corresponding to three projectors $\hat{P}_{\alpha 1}, \hat{P}_{\alpha 2}, \hat{P}_{\alpha 3}$ in QM) and put them on the optical bench in pairs. Assume that there are three random variables $P_i(w)$ that reflect the QM correlations, i.e. $\mu(P_i = 1, P_j = 1) = ½ \cos^2(\alpha_i - \alpha_j)$. Then

$$\mu(P_i = 1, P_j = 0) = \mu(P_i = 1) - \mu(P_i = 1, P_j = 1) =$$
$$½ - ½ \cos^2(\alpha_i - \alpha_j) = ½ \sin^2(\alpha_i - \alpha_j).$$

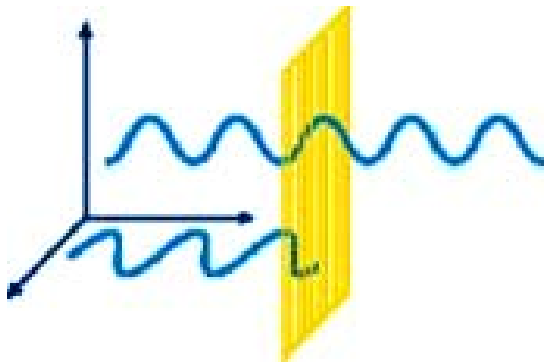
Then Bell's third inequality reads

$$½ \sin^2(\alpha_1 - \alpha_3) \le ½ \sin^2(\alpha_1 - \alpha_2) + ½ \sin^2(\alpha_2 - \alpha_3).$$

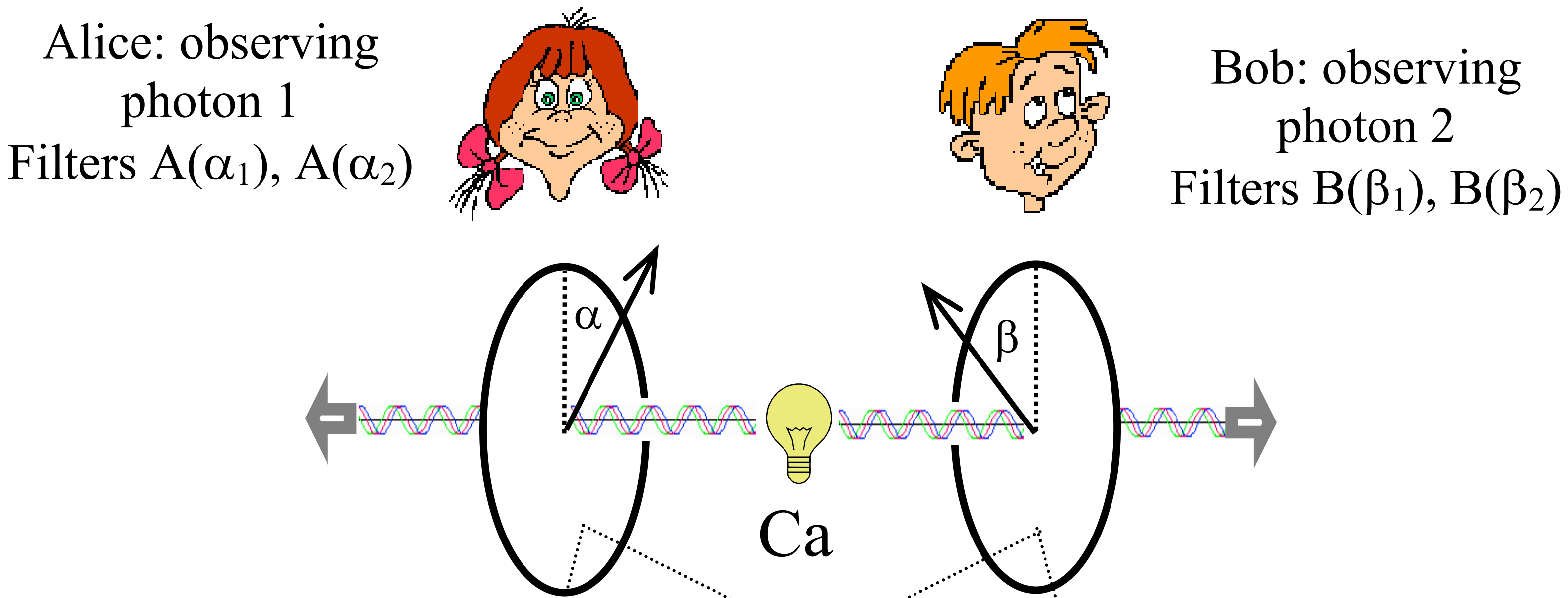
This is wrong for $\alpha_1 = 0$, $\alpha_2 = \pi/6$, $\alpha_3 = \pi/3$: $3/8 > 1/8 + 1/8$!

Counterarguments

- Is there really **one** photon state measured in all three cases? Could not filter *i* influence the photon's reaction to filter *j*?
- In fact, it seems quite obvious that it will. Are there possibilities to avoid this situation?
- Calcium atoms excited by a laser emit pairs of photons. The two photons always have orthogonal polarization.



- In an improved experiment, we should let the filters act on each of the photons without influence on each other.



$$|\psi\rangle = \tfrac{1}{\sqrt{2}}\left(|\leftrightarrow\rangle \otimes |\updownarrow\rangle - |\updownarrow\rangle \otimes |\leftrightarrow\rangle\right)$$
$$= \tfrac{1}{\sqrt{2}}\ (0,\ 1,\ -1,\ 0)$$

Proposition

For any four Boolean (0-1) random variables A_1, A_2, B_1, B_2 on a classical probability space (W, μ) the following inequality holds:

$$\mu(A_1 = B_1) \leq \mu(A_1 = B_2) + \mu(A_2 = B_1) + \mu(A_2 = B_2)$$

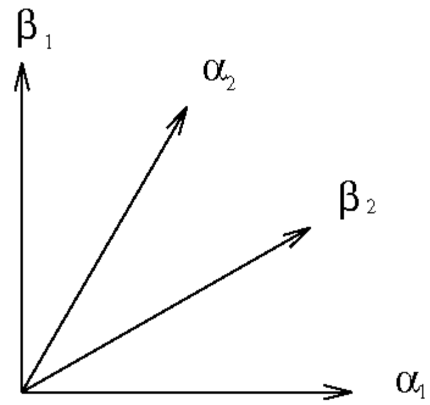
Proof

Write C_{ij} for the random variable which is 1 if $A_i = B_j$ and 0 otherwise. It is obvious then that $\mu(A_i = B_j)$ iff $\mu(C_{ij}) = 1$.

It is simply to show now that $C_{11} \leq C_{12} + C_{21} + C_{22}$. The contrary would mean that for some possible world $w \in W$, $C_{11}(w) = 1$ and $C_{12}(w) = C_{21}(w) = C_{22}(w) = 0$; i.e. $A_1(w) = B_1(w)$ and $A_1(w) \neq B_2(w) \neq A_2(w) \neq B_1(w)$. This is not possible because there are an odd number of inequality signs here. From the inequality $C_{11} \leq C_{12} + C_{21} + C_{22}$ the conclusion is an immediate consequence.

A violation of Bell's fourth inequality

QM predicts $\mu(A_i = B_j) = \frac{1}{2}\sin^2(\alpha_i - \beta_j)$ if we consider the pure state $|\psi\rangle$ given above. Hence, Bells fourth inequality reads



$$\sin^2(\alpha_1 - \beta_1) \leq \sin^2(\alpha_1 - \beta_2) + \sin^2(\alpha_2 - \beta_1) + \sin^2(\alpha_2 - \beta_2)$$

It is clearly violated for the choices $\alpha_1 = 0$, $\alpha_2 = \pi/3$, $\beta_1 = \pi/2$, and $\beta_2 = \pi/6$, in which case it reads

$$1 \leq \tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{4}$$

Explicit calculations

$$\mu_\psi((\hat{A}_\alpha \otimes \hat{1})(\hat{1} \otimes \hat{B}_\beta)) = \mu_\psi((\hat{A}_\alpha \otimes \hat{B}_\beta)) =$$

$$\langle\psi| \begin{pmatrix} \cos^2\alpha & \cos\alpha\cdot\sin\alpha \\ \cos\alpha\cdot\sin\alpha & \sin^2\alpha \end{pmatrix} \otimes \begin{pmatrix} \cos^2\beta & \cos\beta\cdot\sin\beta \\ \cos\beta\cdot\sin\beta & \sin^2\beta \end{pmatrix} |\psi\rangle$$

$$= \tfrac{1}{2}\,(0, 1, -1, 0) \begin{pmatrix} \cos^2\alpha\cos^2\beta & \cos^2\alpha\cos\beta\sin\beta & \cos\alpha\sin\alpha\cos^2\beta & \cos\alpha\sin\alpha\cos\beta\sin\beta \\ \cos^2\alpha\cos\beta\sin\beta & \cos^2\alpha\sin^2\beta & \cos\alpha\sin\alpha\cos\beta\sin\beta & \cos\alpha\sin\alpha\sin^2\beta \\ \cos\alpha\sin\alpha\cos^2\beta & \cos\alpha\sin\alpha\cos\beta\sin\beta & \sin^2\alpha\cos^2\beta & \sin^2\alpha\cos\beta\sin\beta \\ \cos\alpha\sin\alpha\cos\beta\sin\beta & \cos\alpha\sin\alpha\sin^2\beta & \sin^2\alpha\cos\beta\sin\beta & \sin^2\alpha\sin^2\beta \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

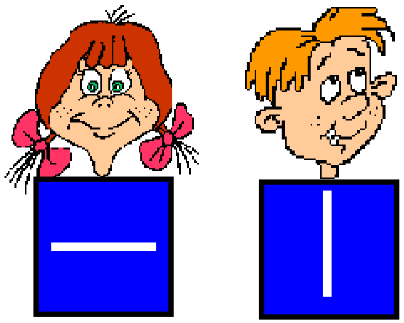
$$= \frac{1}{2}(\cos^2\alpha\sin^2\beta + \sin^2\alpha\cos^2\beta - 2\cos\alpha\sin\alpha\cos\beta\sin\beta)$$

$$= \frac{1}{2}(\cos\alpha\sin\beta - \sin\alpha\cos\beta)^2$$

$$= \frac{1}{2}\sin^2(\alpha-\beta)\quad .$$

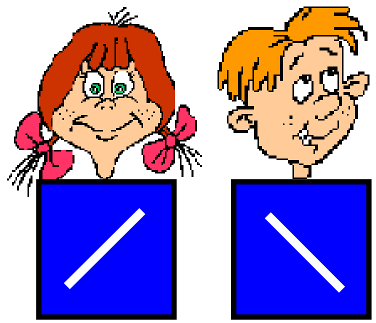
Some remarks

- The state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\leftrightarrow\rangle \otimes |\updownarrow\rangle - |\updownarrow\rangle \otimes |\leftrightarrow\rangle)$ cannot be written as the outer product $|u\rangle \otimes |v\rangle$ of two vectors of the corresponding subspaces (entanglement)
- It is surprising that the correlation $\mu_\psi(\hat{A}_\alpha \otimes \hat{B}_\beta)$ depends on the difference $\alpha-\beta$ of the angles only.



Same correlations *Corr* = ½

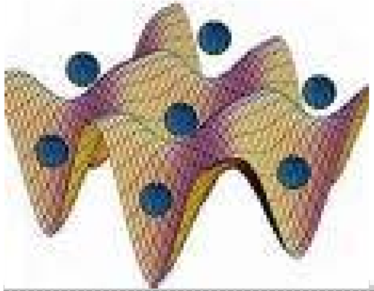
$$\frac{1}{\sqrt{2}}(|\leftrightarrow\rangle \otimes |\updownarrow\rangle - |\updownarrow\rangle \otimes |\leftrightarrow\rangle)$$



- $\frac{1}{\sqrt{2}}(|\leftrightarrow\rangle \otimes |\updownarrow\rangle - |\updownarrow\rangle \otimes |\leftrightarrow\rangle) = \frac{1}{\sqrt{2}}(|\nearrow\rangle \otimes |\searrow\rangle - |\searrow\rangle \otimes |\nearrow\rangle)$

Conclusions

- Is there a representation of states that has room for every "putative state", i.e. a consistent description of probabilities for an orthoalgebra of projections. Gleason's famous theorem says yes: take the density matrix.
- The violations of Bell's inequalities demonstrate that the formalism of quantum theory does not admit of a certain sort of hidden variable interpretation.
- Quantum entanglement is a quantum mechanical phenomenon in which the quantum states of two or more spatially separated objects can apparently have an instantaneous influence on one another. This effect is now known as "nonlocal behaviour".



5 Aspects of quantum computation

Prof. Sham: Well, we factored number 15 into 3 times 5. It was a thorny problem, but, by Jove, we did it. We started building our NMR computer in 1997 with a 5-million-dollar grant from DARPA. So it took 5 years, a few million dollars, and much hard work to build our room sized computer. But just think how powerful it is! And according to theory, NMR computers have tremendous growth potential. They can have as many as 10 qubits!

Reporter: Could you please tell us something about your future plans?

Prof. Sham: Well. I plan to continue publishing in Nature Magazine. Another exciting development is that I plan to add one more qubit to NMR computers in the next few years. We've won another 3-million-dollar grant from DARPA to continue our work. Isn't our government just great! We can't wait to start factoring number 18.

Single Q-bit gates

X-gate: $\hat{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ transforms $|\updownarrow\rangle$ in $|\leftrightarrow\rangle$ and $|\leftrightarrow\rangle$ in $|\updownarrow\rangle$

Negation: take $|\updownarrow\rangle$ as ***0*** and $|\leftrightarrow\rangle$ as ***1***

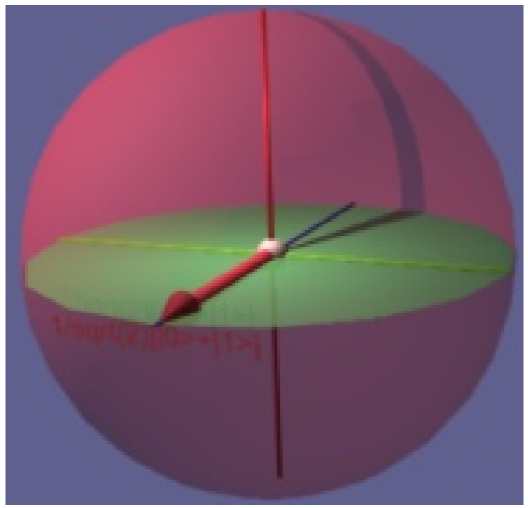
Z-gate: $\hat{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ leaves $|\updownarrow\rangle$ unchanged,

turns $|\leftrightarrow\rangle$ into $-|\leftrightarrow\rangle$

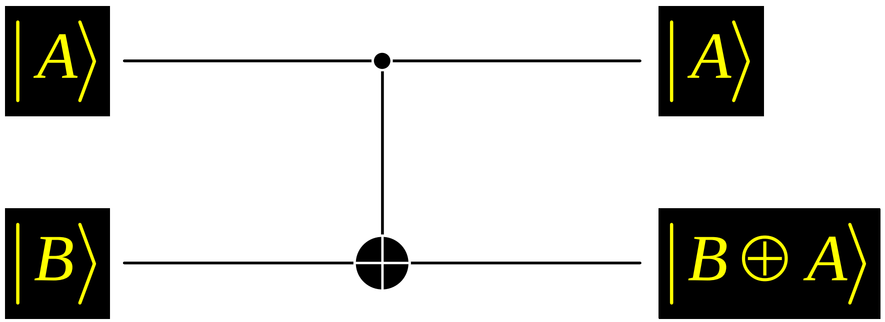
Hadamard Gate $\hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ turns $|\updownarrow\rangle$ into $|\nearrow\rangle$ and

turns $|\leftrightarrow\rangle$ into $|\searrow\rangle$

All these gates realize unitary transformations.
Note that $\hat{H} = \frac{1}{\sqrt{2}}(\hat{X} + \hat{Z})$



Controlled NOT gate



$$U_{CN} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

If control is $|\updownarrow\rangle$ target left alone $|\updownarrow\updownarrow\rangle$ in $|\updownarrow\updownarrow\rangle$ or $|\updownarrow\leftrightarrow\rangle$ in $|\updownarrow\leftrightarrow\rangle$;
else control is $\leftrightarrow$ target Q-bit is flipped $|\leftrightarrow\updownarrow\rangle$ in $|\leftrightarrow\leftrightarrow\rangle$
or $|\leftrightarrow\leftrightarrow\rangle$ in $|\leftrightarrow\updownarrow\rangle$.

Any multiple Q-bit logic gate may be composed from CNOT and single Q-bit Gates

A two Q-bit quantum computer

It is able to realize an arbitrary function f(x): $\{0,1\} \rightarrow \{0,1\}$

1. Realize a unitary operator U_f such that $U_f(|x,y\rangle)=|x,y\oplus f(x)\rangle$, where $\oplus$ denotes addition modulo 2
2. Consider the state $|\psi\rangle = \frac{1}{\sqrt{2}}(\,|\updownarrow,\updownarrow\rangle+|\leftrightarrow,\updownarrow\rangle)$ and apply U_f
3. $U_f(\,|\psi\rangle\,) = \frac{1}{\sqrt{2}}(\,|\updownarrow,f(\updownarrow)\rangle+|\leftrightarrow,f(\leftrightarrow)\rangle)$.

   This is an entangled state that realizes (in parallel) the wanted computation. (Both values of the function show up in the final state solution)
4. This can be generalized to functions on arbitrary number of bits using the Hadamard transform.

Break through

- Deutsch (1985) proposed the simplest example of a quantum algorithm which outperforms a classical algorithm.
  - The algorithm has given us the ability to determine a GLOBAL PROPERTY of f(x), namely $f(0) \oplus f(1)$ using only ONE evaluation of f(x)
  - A classical computer would require at least two evaluations!
- Shor (1994) developed a quantum algorithm which can factorize efficiently.
- Other quantum algorithms concern search problems and the simulation of quantum systems.

6 Applications in the macroworld

Three conceptions of applying QM to the macroworld

(A) Reductionist approach

Understand phenomena such as complementarity, entanglement, consciousness etc. always as manifestation of QM micro-laws

(B) Quantum Computation

Engineering approach of friezing short-living quantum states

(C) Systemic Approach

Some structures of the micro world that are investigated by QM appear in non-physical domains

Representatives of the systemic approach

Diederik Aerts, Liane Gabora.

"While some of the properties of quantum mechanics are essentially linked to the nature of the microworld, others are connected to fundamental structures of the world at large and could therefore in principle also appear in other domains than the micro-world."

Harald Atmanspacher, Hans Primas, Peter beim Graben.

"A generalized version of the formal scheme of ordinary quantum theory, in which particular features of ordinary quantum theory are not contained, should be used in some non-physical contexts."

Andrei Khrennikov

"According to our interference viewpoint to quantumness human being is quantum – but not because it is composed of microscopic quantum systems."

Potential applications

- Puzzles of bounded rationality: interference effects
- Prototype semantics and conceptual combination: entanglement
- Theory of ambiguities: superposition vs. mixture
- Nonclassical theory of questions and answers: uncertainty principle (cf. the psychology of C.G. Jung)
- Modelling connectionist networks: tensor product.

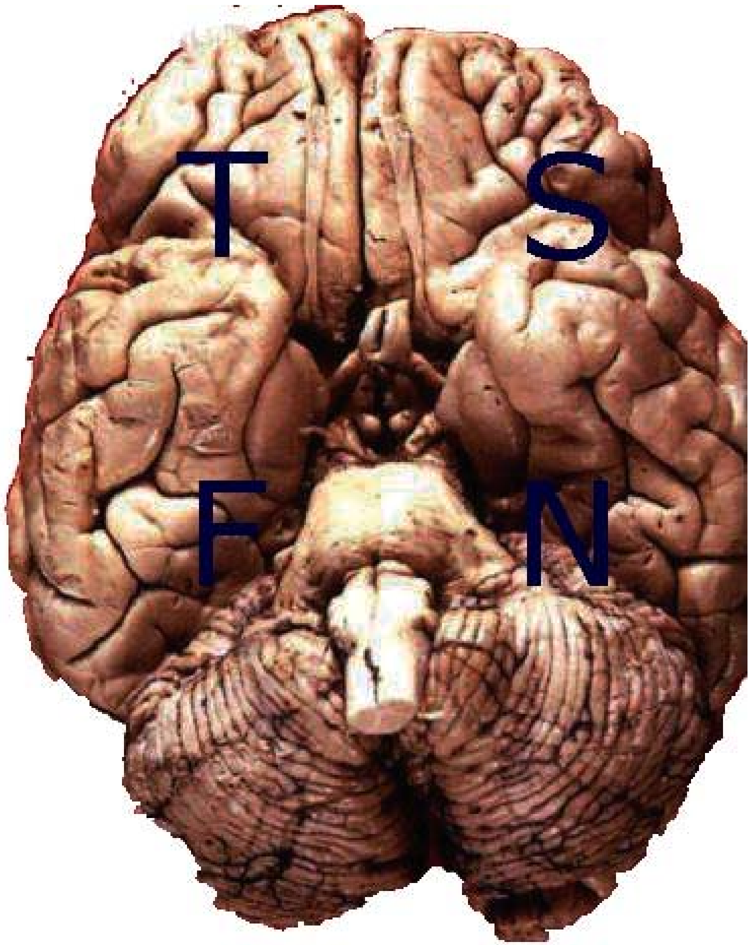
7 Two qubits for Jung's theory of personality

C.G. Jung's theory of personality in a nutshell

- All people have broadly the same psychological equipment of apperception and responsiveness
- Where people differ is the way that each of them typically makes use of the equipment
- Main questions for the psychologist:
  - What are the essential components of the equipment?
  - How do people differ in using these components to form their habitual mode of adaptation to reality?

The four psychological attitudes

- Two rational opposites:
  –**T**hinking / **F**eeling (evaluation)
- Two irrational opposites:
  –**S**ensation (perception) / i**N**tuition
- ***S**ensation* tells us that something exist; ***T**hinking* tells you what it is; ***F**eeling* tells you whether it is agreeable or not; and *i**N**tuition* tells you whence it comes and where it is going (Man and his Symbols, 61)



The two attitudes

- **E**xtravert → primarily oriented to events in the outer world
- **I**ntrovert → primarily concerned with the inner world

- The functions are always realized under a certain attitude, either extroverted or introverted

- We all have these four psychological functions (in one or the other attitude). We just have them in different proportions.

Some test questions

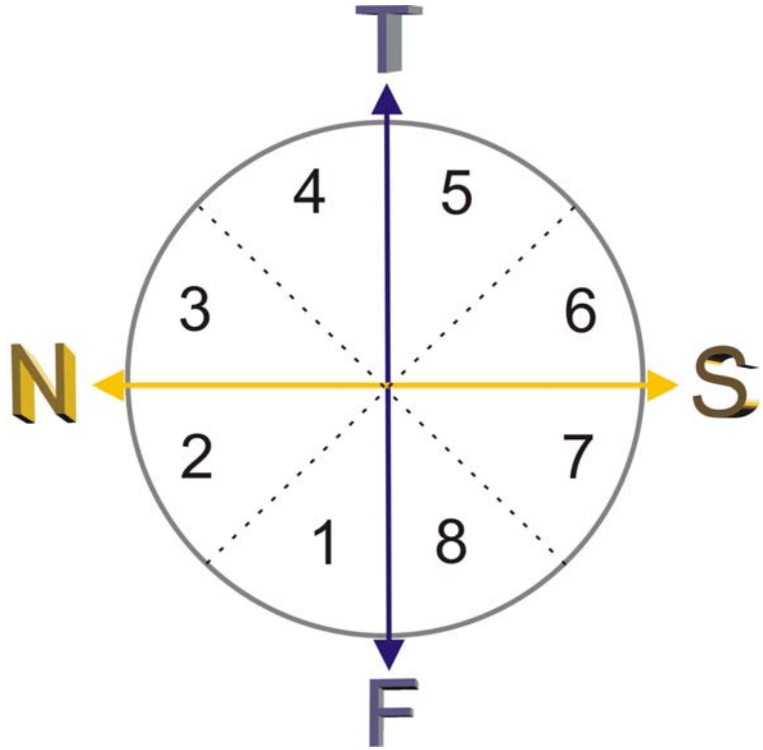
(1) **E**xtraverted/**I**ntroverted opposition
   a. At parties, do you stay late with increasing energy or leave early with decreased energy? (E/I)
   b. When the phone rings, do you hasten to get to it first, or do you hope someone else will answer? (E/I)

(2) **F**eeling/**T**hinking opposition
   a. In making decisions do you feel more comfortable with feelings or standards? (F/T)
   b. In order to follow other people do you need trust, or do you need reason? (F/T)

(3) **S**ensing/i**N**tuition opposition
   a. Which seems the greater error: to be too passionate or to be too objective? (S/N)
   b. Facts speak for themselves or illustrate principles? (S/N)

C.G. Jung’s eight basis types

E/I **F**eeling type 1 & 8
E/I i**N**tuition type 2 & 3
E/I **T**hinking type 4 & 5
E/I **S**ensing type 6 & 7



These eight basic types discussed by Jung can be further refined into 16 psychological types depending on what is considered as the secondary function (E/I 1-8)

Two Restrictions

Assume that the relevant attitude-function pairs are linearly ordered. Then the following restrictions apply:

- If the superior function is rational/irrational, then the secondary function must be irrational/rational. This alternation of rational and irrational function is continued along the ranking hierarchy.

- Opponent (or *dual*) functions (i.e. T/F and N/S) have different attitudes (otherwise, they could not 'coexist')

Socionics & MBTI: 16 Types

16 psychological types
The first two dominant psychological functions are given with the corresponding attitude (extraverted / introverted). Further, the closest pendant in the MBTI is specified. E.g.

C.G. Jung 1IN 2IT
S. Holmes 1ET 2ES
Leo Tolstoi 1EN 2EF
G.I Cäsar 1ES 2EF Napoleon 1ES 2EF

	Extravert		Introvert		
1	1EF 2EN	ENFJ	1IF 2IN	INFP	1
2	1EN 2EF	ENFP	1IN 2IF	INFJ	2
3	1EN 2ET	ENTP	1IN 2IT	INTJ	3
4	1ET 2EN	ENTJ	1IT 2IN	INTP	4
5	1ET 2ES	ESTJ	1IT 2IS	ISTP	5
6	1ES 2ET	ESTP	1IS 2IT	ISTJ	6
7	1ES 2EF	ESFP	1IS 2IF	ISFJ	7
8	1EF 2ES	ESFJ	1IF 2IS	ISFP	8

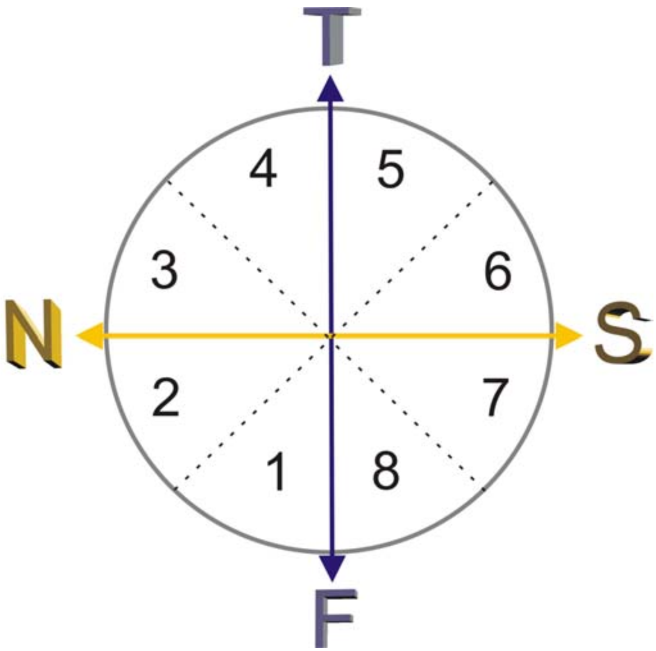
Opinion forming

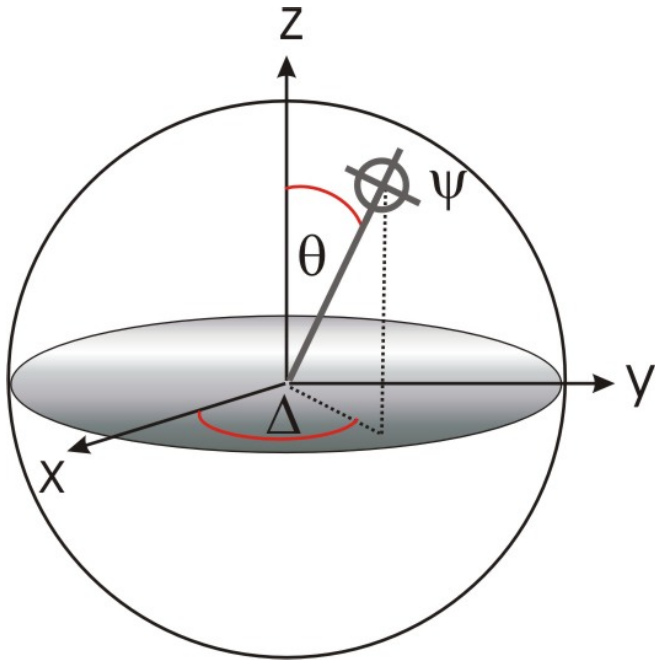
Q1 : Are you in favour of the use of nuclear energy?
Q2 : Do you think it would be a good idea to legalize soft-drugs?
Q3 : Do you think capitalism is better than social-democracy?

Interestingly, in such situations most people don't have a predetermined opinion. Instead, the opinion is formed to a large extend during the process of questioning in a context-dependent way. That means, opinions formed by earlier questions can influence the actual opinion construction.

Observables for T, F, S, and N

$|\psi\rangle = \cos(\theta/2)\,|0\rangle + \sin(\theta/2)\,|1\rangle$ (Zero phase shift Δ)



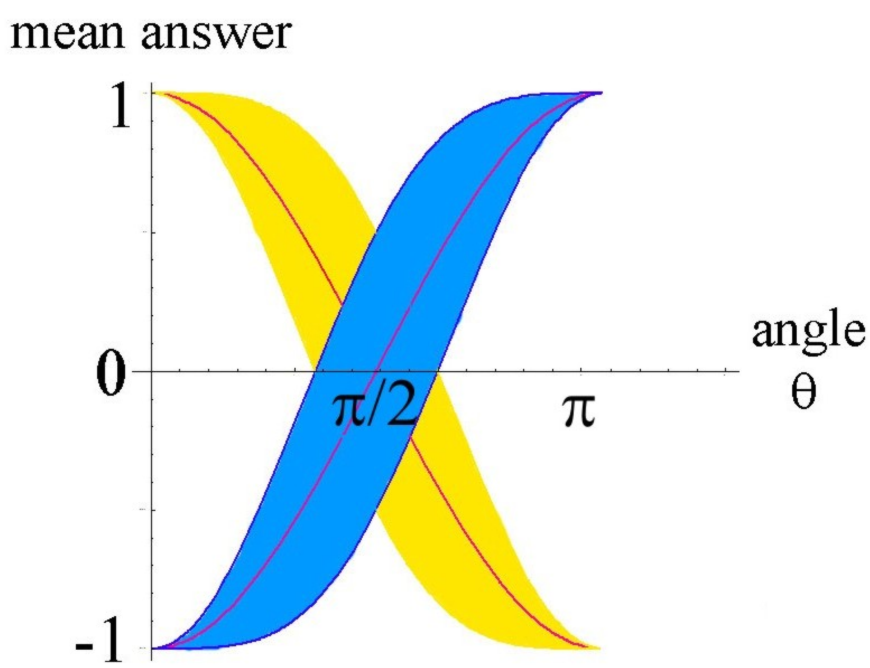


$T = \hat{Z}$, $F = -\hat{Z}$, $S = \hat{X}$, $N = -\hat{X}$

Expectation values for the opponent observables

$\hat{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $-\hat{Z} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

in case of a qubit state with zero phase shift Δ including an indication of the corresponding standard derivations.

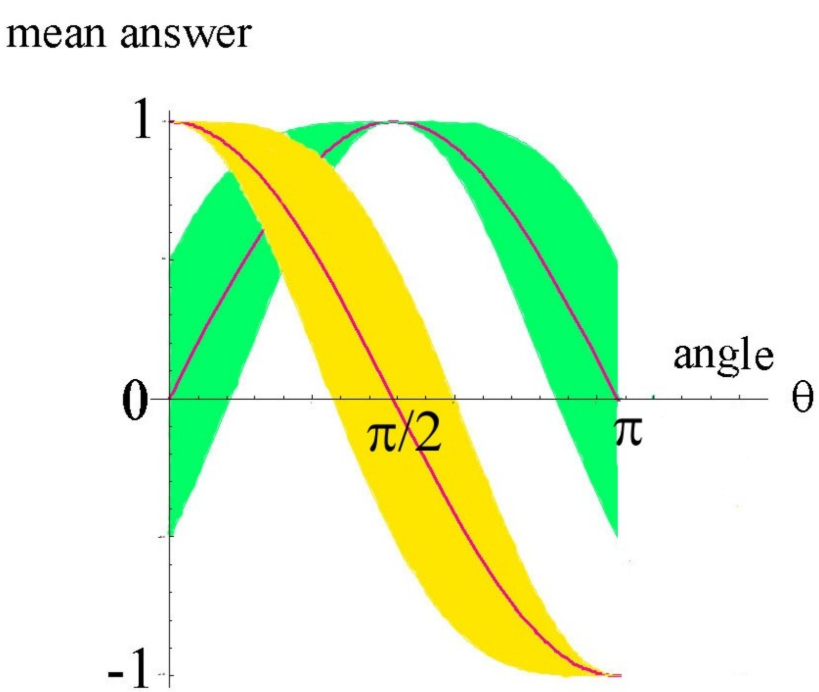


Complementary observables

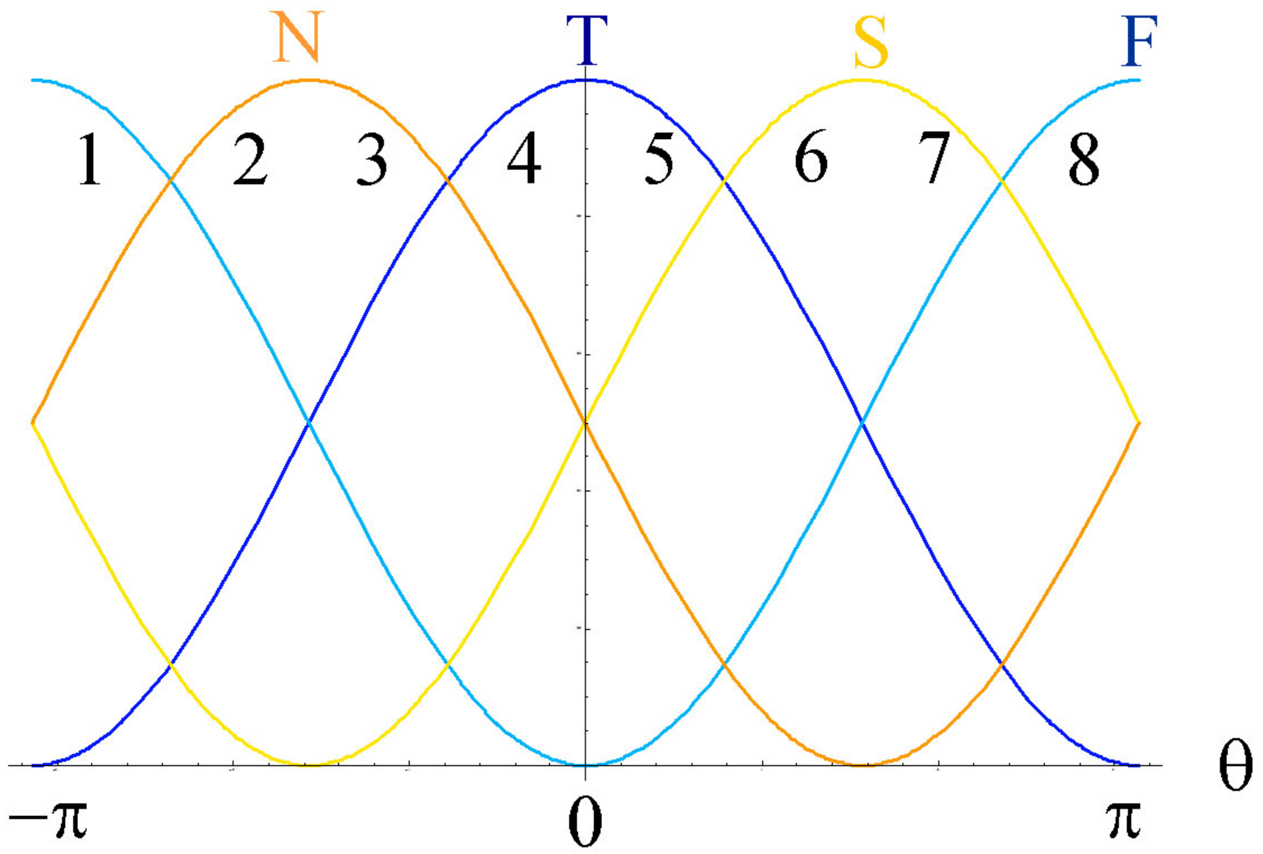
Expectation values for the complementary observables

$$\hat{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad \hat{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

in case of a qubit state with zero phase shift Δ including an indication of the corresponding standard derivations.



The 8 types



1. F>N>S>T
2. N>F>T>S
3. N>T>F>S
4. T>N>S>F
5. T>S>N>F
6. S>T>F>N
7. S>F>T>N
8. F>S>N>T

The 8 types as resulting from different proportions of the expectation values for N, T, S, and F.

- Opponent (or *dual*) psychological functions (i.e. T/F and N/S) are realized with contrasting attitudes.
- In the present formal theory, this idea suggests that the attitudes are entangled with the psychological functions
- $|\Psi\rangle = |\alpha\rangle \otimes |\psi\rangle - |\alpha\rangle^{\perp} \otimes |\psi\rangle^{\perp}$

  Here the first qubit stands for the E/I dimension and the second for the psychological functions
- $C_\Psi(E,T)+C_\Psi(M,T)+C_\Psi(E,S)-C_\Psi(M,S) = 2\sqrt{2} > 2$ (Bell violation!) M = interMediate (between E and I).

Calculating the expected answers

- $|\Psi\rangle = |0\rangle \otimes |\psi\rangle - |1\rangle \otimes |\psi\rangle^{\perp}$ ($|0\rangle$ stands for E/I)

- $$\begin{array}{llll} \langle T / E = 1\rangle_\psi = \cos(\theta) & & \langle F / I = 1\rangle_\psi & = \cos(\theta) \\ \langle S / E = 1\rangle_\psi = \sin(\theta) & & \langle N / I = 1\rangle_\psi & = \sin(\theta) \\ \langle N / E = 1\rangle_\psi = -\sin(\theta) & & \langle S / I = 1\rangle_\psi & = -\sin(\theta) \\ \langle F / E = 1\rangle_\psi = -\cos(\theta) & & \langle T / I = 1\rangle_\psi & = -\cos(\theta) \end{array}$$

- Consider region 5 as example, i.e. take $0 < \theta < \pi/4$: then we get the ranking 1T 2S 3N 4F for the extraverted attitude, and for the inverted attitude we get the ranking 1F 2N 3S 4T.

Fitting personality types

Subjects have to answer the questions conforming to E, I, T, F, ET, IT, … Parameter fitting based on maximum likelihood.

Two parameter model: include θ and ρ (relative strength of the two parts of the entangled state $|\Psi\rangle$.

So far we have assumed $\rho = 1$, i.e. both parts are equally strong (reflecting the case of an "ideal" personality integrating its own **shadow** – i.e., it is symmetric under the X-gate operation). According to Jung's idea of *type elaboration*, the process starts without entanglement ($\rho = 0$) and during the process the parameter ρ is slowly increased up to $\rho = 1$.

Three parameter model: add phase angle Δ.

Conclusions

- Quantum theory, as a mathematical construction, provides a natural framework for giving a sound foundation of C.G. Jung's theory of personality.
- It is the abstracted formalism which is ‘borrowed’ from quantum theory, not in any way its microphysical ontology of particles and fields.
- Notions from quantum physics fit better with the conceptual, algebraic and numerical requirements of the cognitive domain than the traditional modelling of concepts in terms of Boolean algebras and the classical probabilities based upon it.
- For more applications see www.quantum-cognition.de